

# 11. Bohm's Theory (Albert Chap 7)

*Topics:*  
I. Principles of BM  
II. BM Explanations of Experiments

**Motivation:** The *state space* of QM has bizarre mathematical properties (in particular, it allows states to be in superpositions). Can we replace it with a state space that (a) is more “classical”; and (b) reproduces the success of QM predictions?

## I. The 4 Principles of Bohmian Mechanics (BM)

1. **State Descriptions:** The state of a physical system is given by **both** a wave function  $\psi$  **and** particle *positions*.

**Recall:** In QM, the state of a physical system is given *entirely* by a wave function  $\psi$  (or, equivalently, a unit vector in a Hilbert space = state space).

In *classical mechanics*, the state of a physical system is given by specifying *positions* and *momenta* ( $p$ ):

For 1 particle, need 6 numbers:  $(x, y, z; p_x, p_y, p_z)$  = a state = 1 point in 6-dim *phase space* = state space.

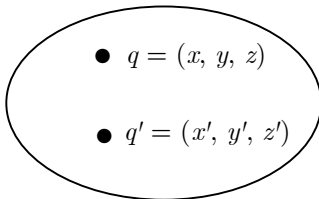
For  $N$  particles, need  $6N$  numbers:  $(x_1, y_1, z_1; p^1_x, p^1_y, p^1_z; \dots ; x_N, y_N, z_N; p^N_x, p^N_y, p^N_z)$ , so have  $6N$ -dim phase space.

**Now:** In *Bohmian mechanics*, a state is given by specifying *positions* and a *wave function*  $\psi$ :

For 1 particle, need  $\psi$  and 3 numbers  $(x, y, z)$  = 1 point in 3-dim *configuration space*.

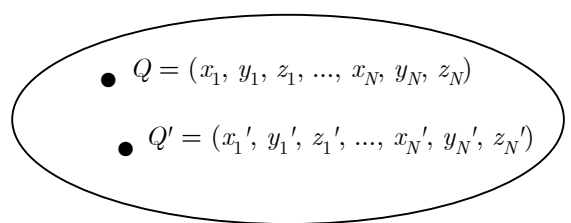
For  $N$  particles, need  $\psi$  and  $3N$  numbers  $(x_1, y_1, z_1; \dots ; x_N, y_N, z_N)$ , so have  $3N$ -dim configuration space.

3-dim configuration space for 1 particle



The point  $q$  represents one possible position of the particle. The point  $q'$  represents another possible position.

3N-dim configuration space for N particles



The point  $Q$  represents one possible position state of the  $N$ -particle system. The point  $Q'$  represents another possible position state.

**Note:** Technically in BM, the state space isn't simply just this configuration space, since states are given by *both* positions *and* a wave function. The configuration space *just* gives you positions. The idea is that the wave function is going to contribute to the dynamics on this configuration space; *i.e.*, it's going to tell you how points in configuration space evolve into other points.

2. **Wave Function Dynamics:** The wave function associated with a state evolves according to the *Schrödinger dynamics*:

$$\psi(Q, t_i) \xrightarrow[\text{Schrödinger evolution}]{t_i \rightarrow t_f} \psi(Q, t_f)$$

3. **Particle Dynamics (Bohm's Equation):** Particle velocities are determined by the wave function *via Bohm's Equation*:

$$\vec{V}_i[\psi(Q)] = \frac{d\vec{q}_i}{dt} = \frac{\hbar}{m_i} \text{Im} \left( \frac{\psi^* \vec{\partial}_i \psi}{\psi^* \psi} \right) \Bigg|_Q$$

The velocity  $\vec{V}_i$  of the *i*th particle (located at  $q_i = (x_i, y_i, z_i)$ ) ...

... is a function of its mass  $m_i$  and the *N*-particle wave function  $\psi(Q)$  (which depends on the positions  $Q$  of all the *N* particles).

**Aside:** The “Im” indicates that we’re supposed to take the imaginary part of the term in brackets.  $\psi^*$  is the complex conjugate of  $\psi$ . If we note that  $\psi^* \psi = |\psi|^2$ , then the right-hand-side of Bohm’s equation is essentially the “probability current”  $\vec{J}_i = (\hbar/m_i) \text{Im}(\psi^* \vec{\partial}_i \psi)$  divided by the “probability density”  $\rho = |\psi|^2$ . So the velocity is essentially defined as the current divided by the density:  $\vec{V}_i = \vec{J}_i / \rho$ . This is analogous to the case in electrodynamics in which the velocity of a charge is defined as the charge current divided by the charge density.

4. **The “Distribution” (or “Statistical”) Postulate:** At *some* time  $t_0$ , particle positions are given by a probability defined by the wave function at  $t_0$ :

$$\text{Pr}(\text{particle positions are } Q \text{ at time } t_0) = |\psi(Q, t_0)|^2$$

**What this entails:**

*BM* reproduces all the *QM* probability predictions! (*BM* is empirically indistinguishable from *QM*.)

**QM says:**

(*Born Rule*) The probabilities for particle positions at **any** time  $t$  are given by  $|\psi(Q, t)|^2$ .

**BM says exactly the same thing, because:**

The probability density  $\rho = |\psi|^2$  is *conserved* by the Schrödinger equation (just like charge density is conserved by Maxwell’s equations).

**So:** If at time  $t_0$ , the probabilities are given by  $|\psi(Q, t_0)|^2$  (*BM Distribution Postulate*), then at any future (or past) time  $t$ , the probabilities will be given by  $|\psi(Q, t)|^2$  (*QM Born Rule*).

**Aside:** Technically, the probability density  $\rho$  satisfies the *equation of continuity* (charge conservation:  $\partial\rho/\partial t + \text{div} J = 0$ , where  $J = (\hbar/m) \text{Im}(\psi^* \partial \psi)$  is the probability current.

What Principles 2, 3, and 4 are saying:

The point  $Q$  (representing the positions of all the  $N$  particles at any given time) moves about configuration space by being “guided” by the wave function  $\psi$ !

$$Q \xrightarrow[\substack{\text{particle} \\ \text{dynamics} \\ \text{via } \psi}]{t_i \rightarrow t_f} Q'$$

One interpretation: The particles are swept along by the probability current defined by  $\psi$ . (The particles are like charges that are swept along in an electrical current.)

BUT: This analogy is not perfect!  $\psi$  only acts in configuration space ( $6N$ -dim for  $N$  particles). It doesn't act in “physical” space. So  $\psi$  literally *isn't* a physical force (like an electric field). But maybe it encodes properties of a physical force.

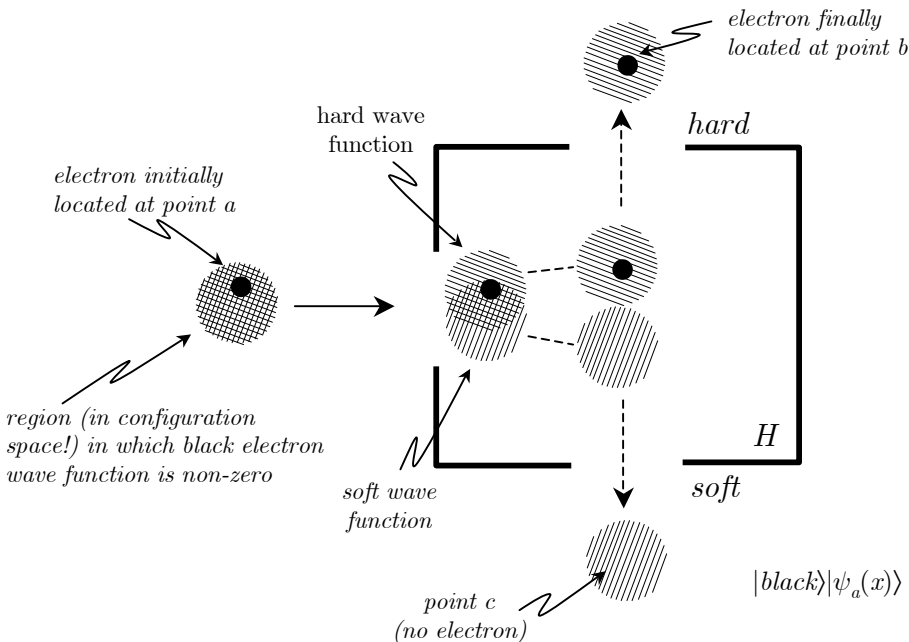
Characteristics of Bohmian Mechanics

- (A) Positions of particles are always determinate. (Particles always have definite positions.)
- (B) Positions evolve completely deterministically. (Any initial position state  $Q$  evolves to a *unique* final position state  $Q'$ .)
- (C) *BM* reproduces the same probability predictions as *QM*.

BUT: In *BM*, probabilities are epistemic! Particles *always* have definite positions, and *BM* probabilities just reflect our ignorance as to what they are.

II. BM Explanations of Experiments

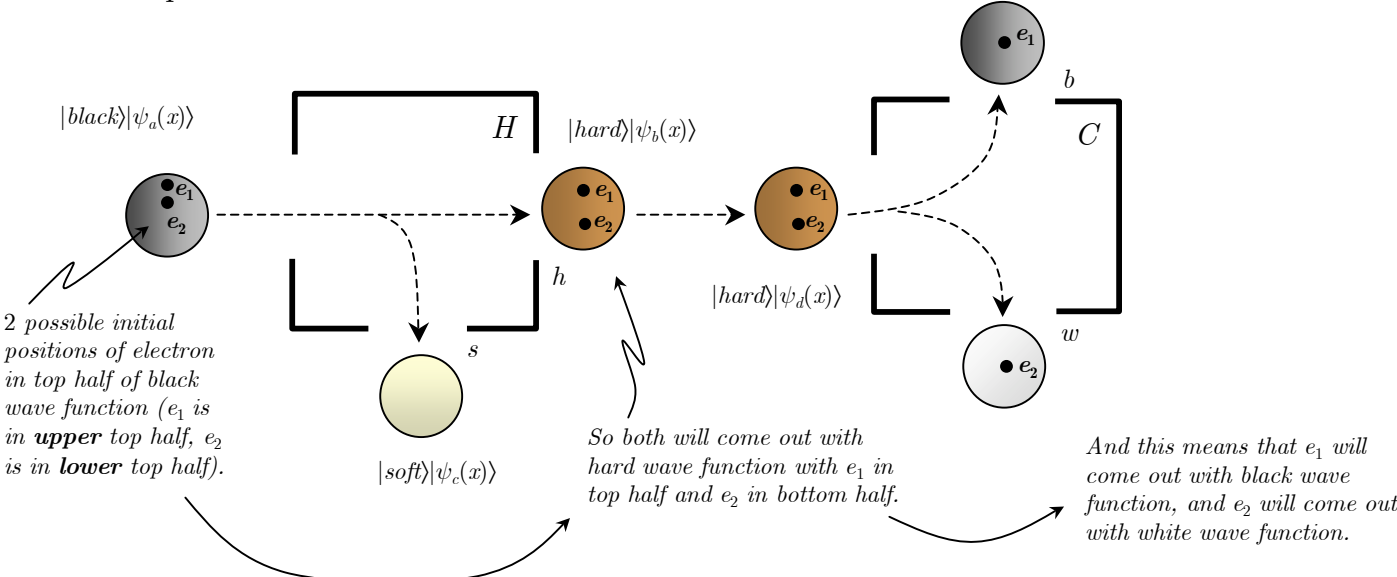
According to Bohm, what happens when we measure the *Hardness* of a *black* electron?



*Black* wave function "splits" into *soft* and *hard* wave functions. Depending on where electron is initially located, it will either be "carried" up with the *hard* wave function, or down with the *soft* wave function (analogy: think of *hard* and *soft* "forces" inside the box that influence where the electron will go). Here, since it's initially located in the upper half of the *black* wave function, it gets carried up (think: *hard* "force" inside box dominates over *soft* "force").

$$|black\rangle|\psi_a(x)\rangle \longrightarrow \sqrt{1/2}|hard\rangle|\psi_b(x)\rangle + \sqrt{1/2}|soft\rangle|\psi_c(x)\rangle$$

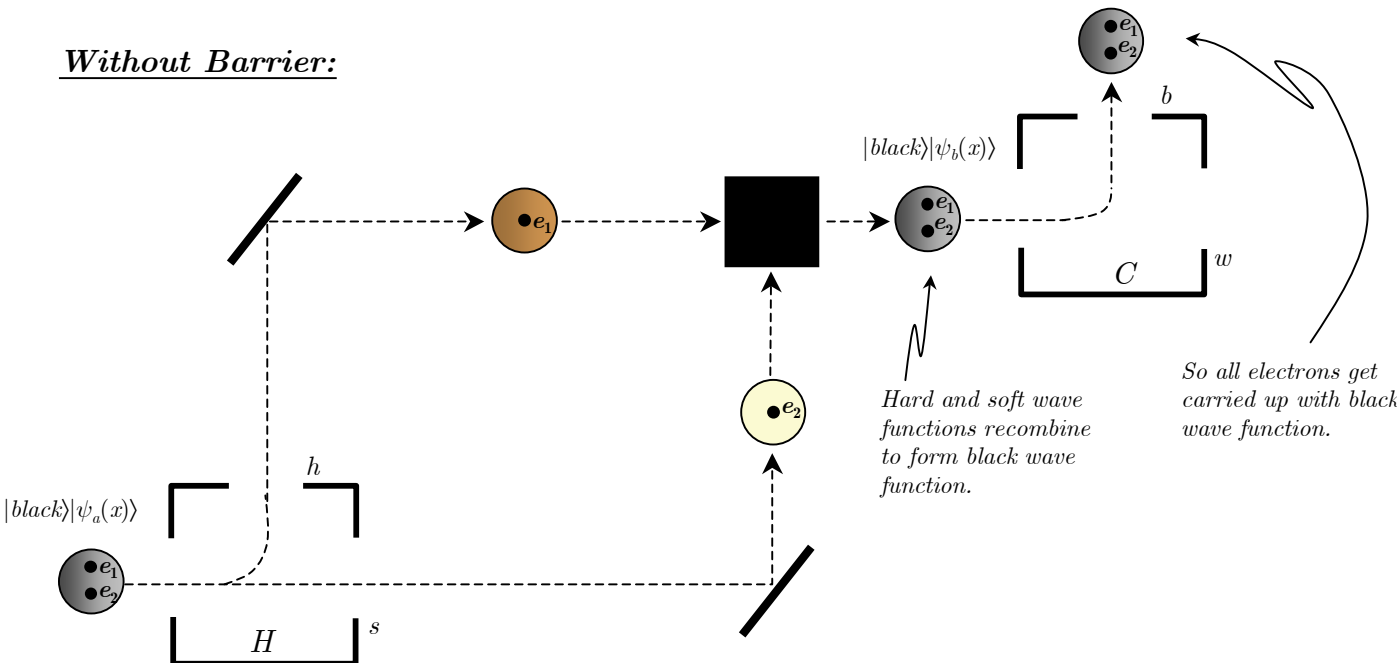
Suppose we start with a *black* electron, first measure its *Hardness*, and then measure its *Color*. Recall original results: Second *Color* measurement gives  $\text{Pr}(\text{black}) = \frac{1}{2}$  and  $\text{Pr}(\text{white}) = \frac{1}{2}$ . Here's how *BM* explains this result:



**SO:** Depending on where the electron is initially located ("in" the initial *black* wave function), it will either come out of the second *Color* box as *black* or as *white*. If it's initially in the top half, then, as the diagram indicates, there's a 50/50 chance that it will emerge as *black* (depending on whether it's in the upper top half or lower top half). And if it's initially in the bottom half of the initial *black* wave function (not indicated in the diagram), then it's pulled down with the *soft* wave function, and when it and its *soft* wave function are then sent through the *Color* box, depending on where it's located in the *soft* wave function, it will also have a 50/50 chance of emerging *black*.

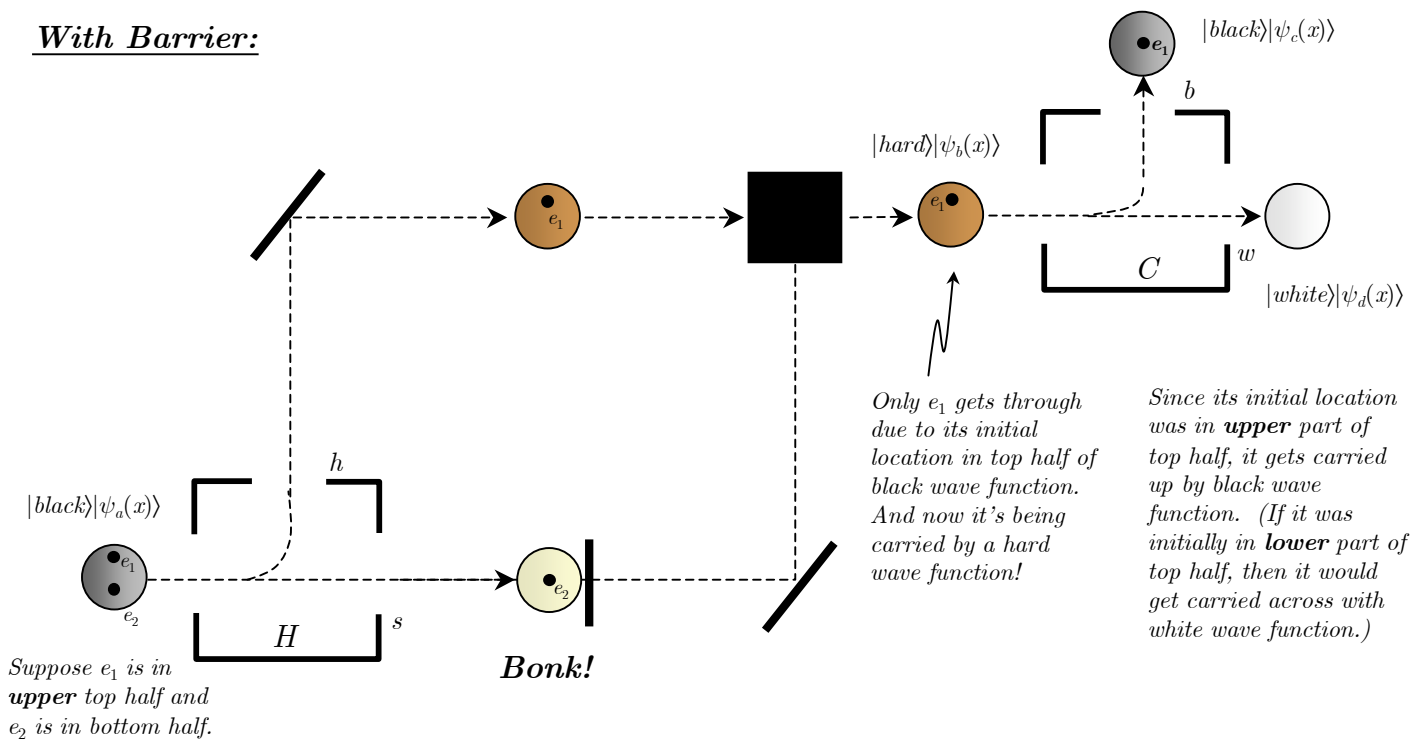
Now let's send *black* electrons through a 2-path device. Recall original results: Without barrier, 100% of electrons exiting will be *black*. With barrier, only 50% get through, and of those, 50% are *black* and 50% are *white*.

Without Barrier:



Suppose  $e_1$  is in top half and  $e_2$  is in bottom half.

With Barrier:



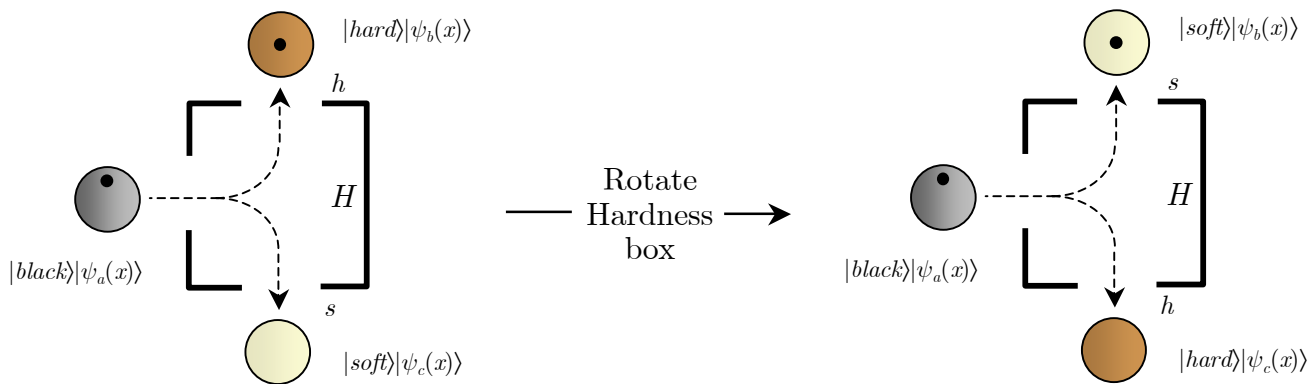
**SO:** Only half of initial *black* electrons get through 2-path box. And of those half, half will go through *black* exit of final *Color* device, and half will go through *white* exit.

Some definitions:

- (a) A property is **intrinsic** just when, whether or not a physical system possesses it does not depend on how it is measured.
- (b) A property is **contextual** just when, whether or not a physical system possesses it depends on how it is measured.

In *BM*, only position is an intrinsic property. All other properties are contextual!

example: In *BM*, *Hardness* is a contextual property.



Electron starts out in same initial location. But, depending on how it is measured, it's *Hardness* value will be either *hard* or *soft*.

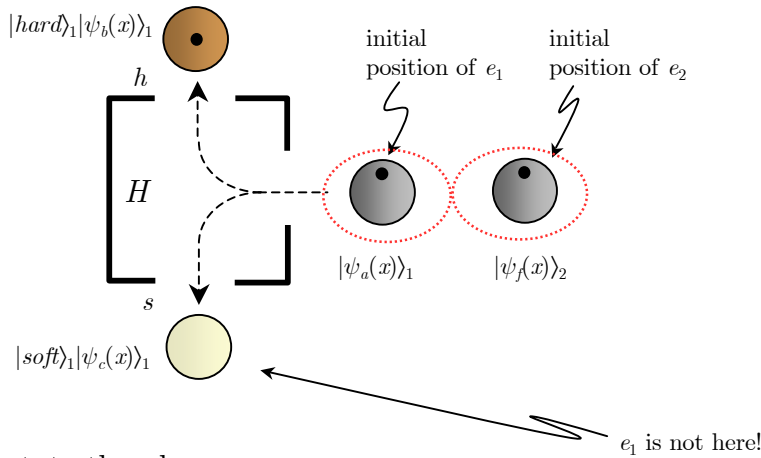
# Bohmian Mechanics and Locality

Suppose we have 2 electrons in an entangled state:

$$\sqrt{1/2} |hard\rangle_1 |\psi_a(x)\rangle_1 |soft\rangle_2 |\psi_f(x)\rangle_2 - \sqrt{1/2} |soft\rangle_1 |\psi_a(x)\rangle_1 |hard\rangle_2 |\psi_f(x)\rangle_2$$

$e_1$  is located in the vicinity of point  $a$ 
 $e_2$  is located in the vicinity of point  $f$

Now measure *Hardness* of  $e_1$ :



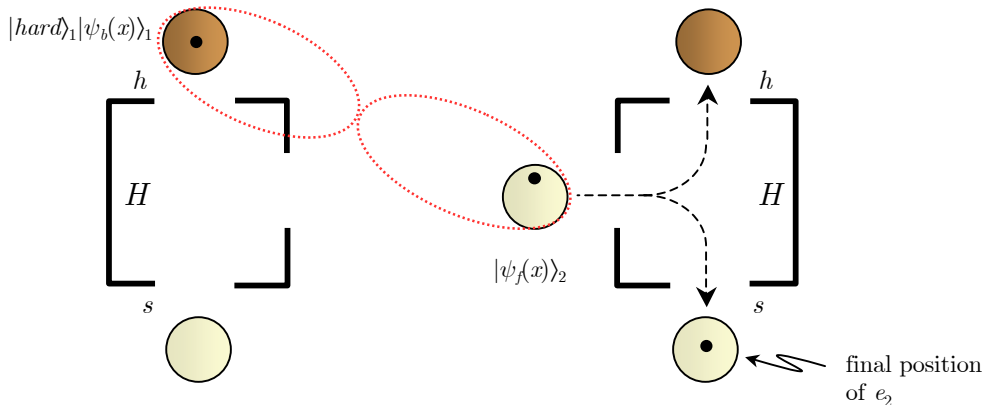
2-particle state then becomes:

$$\sqrt{1/2} |hard\rangle_1 |\psi_b(x)\rangle_1 |soft\rangle_2 |\psi_f(x)\rangle_2 - \sqrt{1/2} |soft\rangle_1 |\psi_c(x)\rangle_1 |hard\rangle_2 |\psi_f(x)\rangle_2$$

0

This part will now influence how  $e_2$  evolves:

Now measure *Hardness* of  $e_2$ :



$e_2$  gets carried down through *soft* exit, because only *soft* wave function acts on it; and this is due to the *prior* measurement of  $e_1$ !

**NOTE:** If  $e_1$  hadn't been measured, then  $e_2$  would have come out *hard*, due to it's initial location!

This type of non-locality is a bit more *spooky* than in *QM*! In *BM*, the electrons *always* have a definite position, and the final position of  $e_2$  is determined by the final position of  $e_1$ . This can be exploited to send faster-than-light signals!

- Suppose  $e_1$  and  $e_2$  are very far apart, and suppose Alice is next to  $e_1$  and Bob is next to  $e_2$ .
- If Bob knows the initial positions of  $e_1$  and  $e_2$  (both in the upper halves of their wave functions, for instance), and he gets the strange result that  $e_2$  came out *soft* (when it should have come out *hard*, given it's initial location), then he *knows* that Alice way over there must have measured  $e_1$  to be *hard*!
- This allows Bob and Alice to send instantaneous signals to each other!

**Example:** Alice desires to send Bob a message instructing him to push either Button *A* or Button *B* at some future time  $t$ . They share initial positions of their  $e_1$  and  $e_2$  and agree to following protocol:

- If Alice wants Bob to push Button *A*, then before  $t$  she orients her *Hardness* box so that a *Hardness* measurement will yield the value *hard*. (This Alice-measurement constitutes the "instantaneous message" being sent.)
- If Alice wants Bob to push Button *B*, then before  $t$  she orients her *Hardness* box so that a *Hardness* measurement will yield the value *soft*.
- At  $t$ , Bob measures his electron (this "decodes" the message Alice just sent him). This will tell him what the outcome of Alice's measurement was, and hence which Button she wants him to push!

**Aside:** Recall under a literal interpretation of *QM*, the outcome of an  $e_2$  measurement depends non-locally on the outcome of an  $e_1$  measurement, **but** the outcome of an  $e_2$  measurement does *not* depend on whether or not an  $e_1$  measurement was done. Bob can never know if the result of his measurement was due to a prior Alice-measurement. This non-locality does not permit signal exchanges. In *BM*, because position is always determinate, the outcome of an  $e_2$  measurement *does* depend on whether or not an  $e_1$  measurement was done. Bob can know whether or not Alice has performed a prior measurement, and he can know exactly what its outcome was.

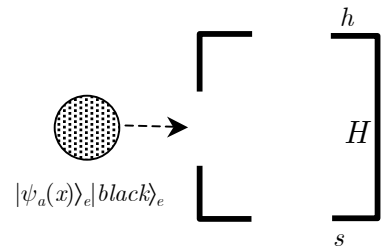
**Two Points:**

1. In special relativity, if two events in spacetime are "space-like separated" (*i.e.*, cannot be connected by the trajectory of an object traveling at the speed of light), then there is *no* absolute fact of the matter which occurs before the other (*simultaneity* is relative to inertial reference frame). But in *BM*, there *will* be a fact of the matter, *if* Alice and Bob can exchange signals. (If they can, then they will always agree on who went first in measuring their respective electrons.) So *BM* will violate special relativity. At the least, *BM* will have to explain why the privileged reference frame that defines absolute simultaneity is in principle unobservable.
2. *BM* can offer such an explanation, and in the process explain why, *in practice*, instantaneous signaling is not possible. Namely, instantaneous signaling will be possible, and special relativity will be violated, **if** initial positions of particles can be known in *BM*. **BUT:**

**Claim:** For any given measurement set-up, the initial positions of particles can *never* be known in *BM*! All that can be known is the wave function.

Why initial particle positions can never be known in BM:

Consider measuring the *Hardness* of a *black* electron  $e$ :



If we could know what  $e$ 's initial position is, then we could predict with certainty which exit it will take:

1. Initially in upper half, then *hard* exit.
2. Initially in lower half, then *soft* exit.

**Note:** *Experimental evidence indicates that we can't know this!*

**Question:** How could we determine initial position?

**Problem:** According to *BM*, any attempt will change the pre-*Hardness* measurement wave function, and so affect all subsequent measurements!

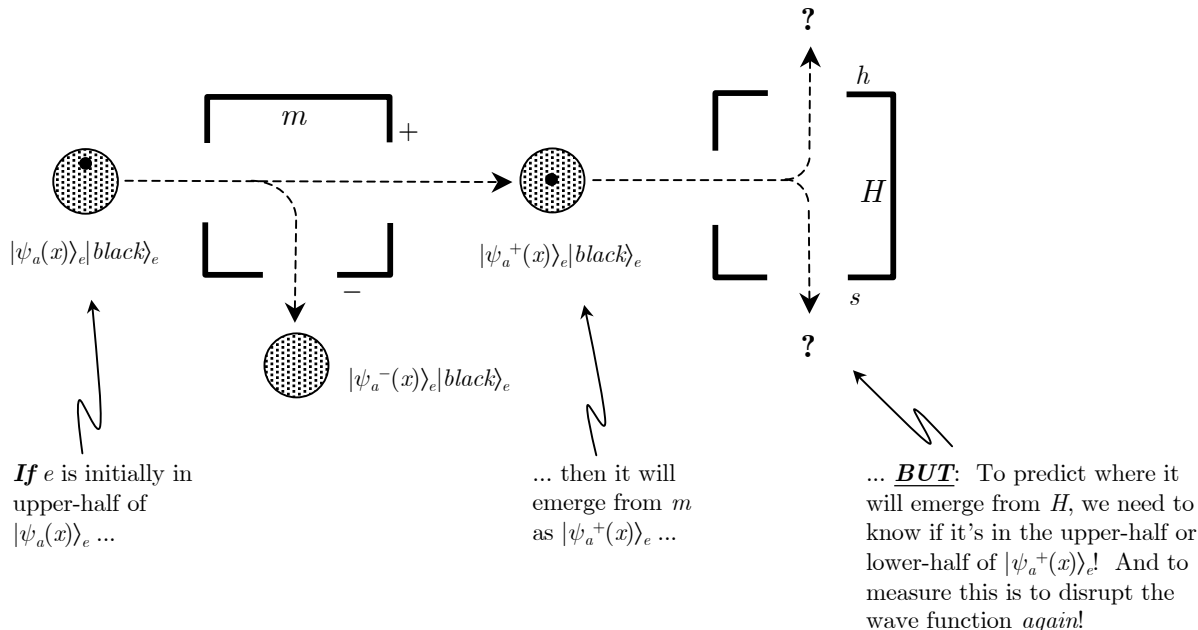
**Example:** Before measuring *Hardness* of  $e$ , measure its position with a Position measuring device  $m$ :

$$|ready\rangle_m |\psi_a(x)\rangle_e |black\rangle_e \longrightarrow \sqrt{1/2} |+\rangle_m |\psi_a^+(x)\rangle_e |black\rangle_e + \sqrt{1/2} |-\rangle_m |\psi_a^-(x)\rangle_e |black\rangle_e$$

$e$  located in upper-half of  $|\psi_a(x)\rangle_e$

$e$  located in lower-half of  $|\psi_a(x)\rangle_e$

If  $e$  is measured to be upper-half of  $|\psi_a(x)\rangle_e$ , say, then all we've done is change it's (effective) wave function from  $|\psi_a(x)\rangle_e$  to  $|\psi_a^+(x)\rangle_e$ . But this will affect how it subsequently evolves! In particular, it will not allow us to predict how it will move through a *Hardness* device.



**Note:** This is *not* to say that particle positions by themselves can never be known in *BM*. If all you want to know are particle positions, then you can set up a position measuring device and get definite results (as in *QM*). What this is saying is that, for any measurement in which positions get correlated with *other* properties, initial positions can never be known.