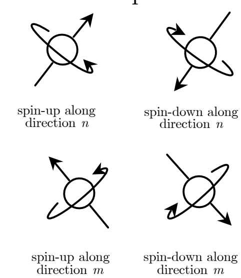
02. The 2-Path Experiment (Albert Chap 1)

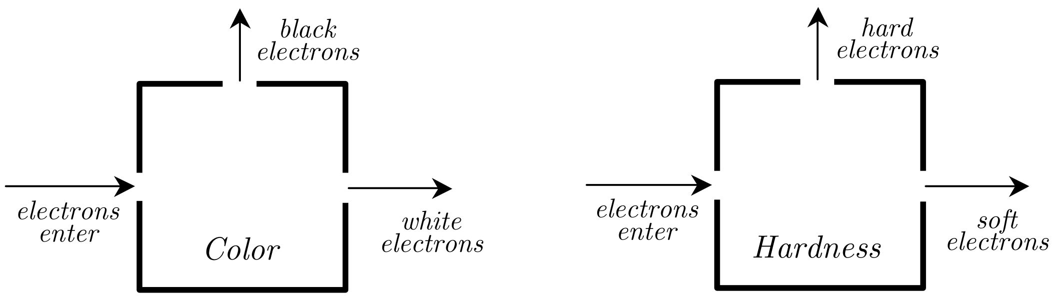
Consider two physical properties of electrons:

"*Color*" -- *black*, *white*
"*Hardness*" -- *hard*, *soft*
} *only 2 possible values*

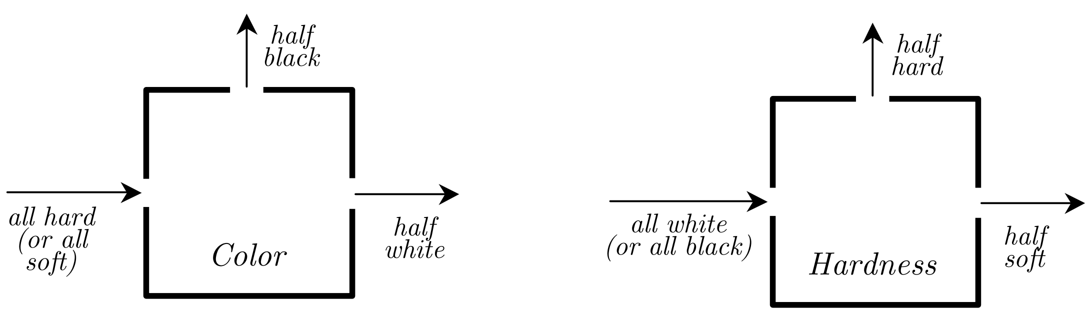
Note: "*Color*" and "*Hardness*" are used by Albert just to make the discussion more concrete. The actual properties being referred to are the *spin* of an electron along two different directions (axes). For electrons, spin along a given direction can have only one of two values ("up" or "down"). All the following results for "*Color*" and "*Hardness*" have been experimentally confirmed for spin.



How to measure *Color* and *Hardness*:

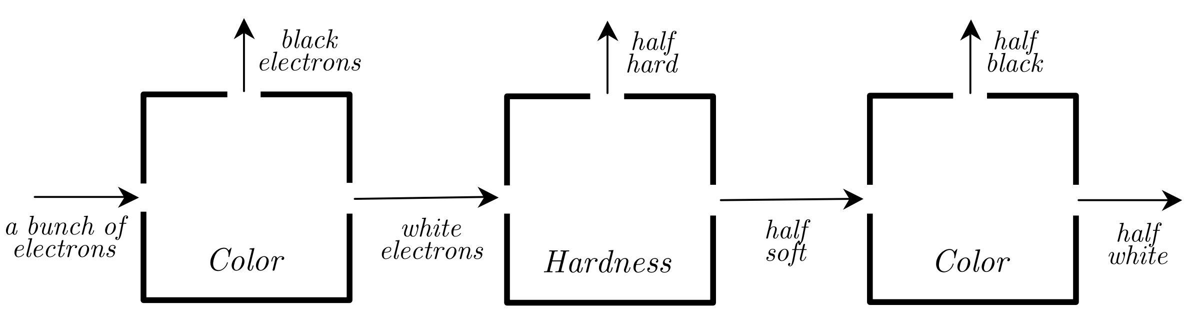


Experimental Result #1: *No correlation between Color and Hardness properties*



Experimental Result #2:

Consider 3 box set-up:



This is ***strange!*** If only white electrons enter the middle *Hardness* box, we'd expect only white electrons to exit the last *Color* box. But they don't!

Does the middle *Hardness* box "disrupt" the color property of the electrons?

Questions:

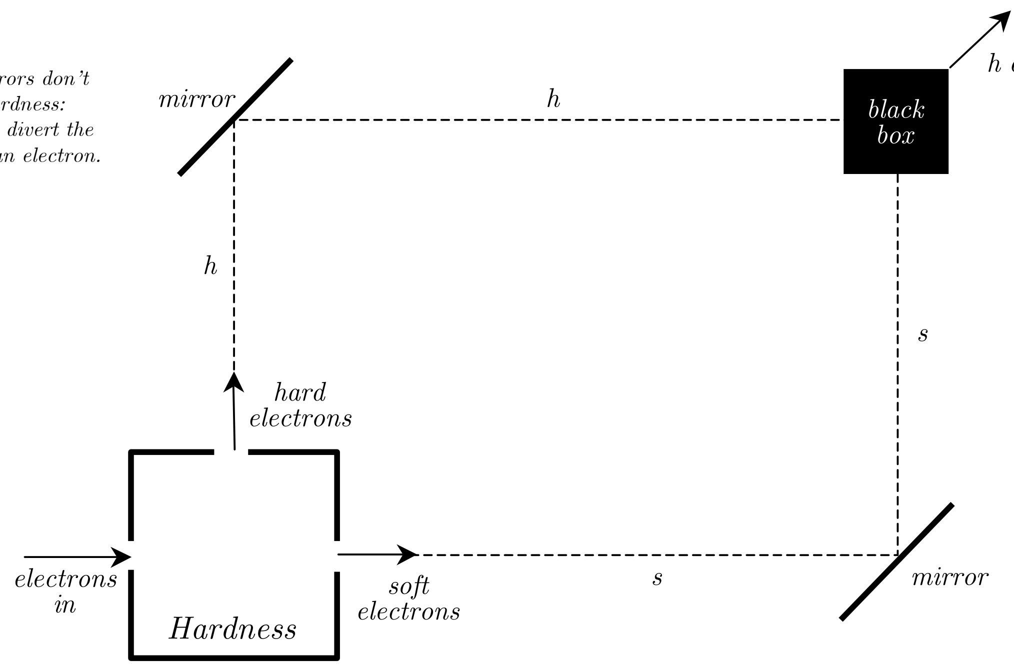
(1) Can we build a *Hardness* box that doesn't disrupt color? ← Experimental evidence says: ***NO!***

(2) Which electrons get their color disrupted by the *Hardness* box? ← Experimental evidence says: ***No way to tell!***

Upshot: Can't simultaneously measure both *Hardness* and *Color*!

Experimental Result #3: *The 2-Path Device* *(essentially a Hardness box)*

The mirrors don't affect hardness: they just divert the path of an electron.



The black box doesn't affect hardness: it just redirects paths h and s to coincide.

Suppose we feed white electrons into the device.

What should we expect to emerge at *h and s*?

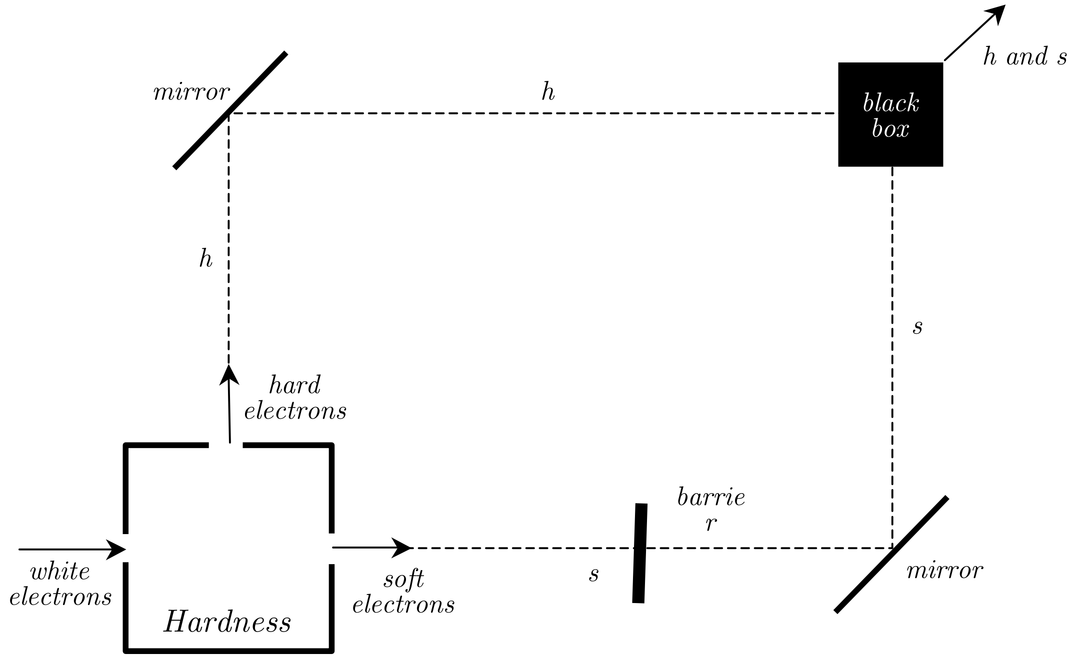
(i) 50% will be *hard* and thus will follow the *h* path. At *h and s*, they will emerge as *hard* electrons, so upon a *Color* measurement, 50% will be *white* and 50% will be *black*.

(ii) 50% will be *soft* and thus will follow the *s* path. At *h and s*, they will emerge as *soft* electrons, so upon a *Color* measurement, 50% will be *white* and 50% will be *black*.

SO: We would expect that, of the initial *white* electrons fed in, 50% will emerge *white* and 50% will emerge *black*. ← *Seems consistent with earlier experiments: The above device is essentially a Hardness box with some mirrors that simply divert the direction of the electrons in it.*

BUT**: **Experiments demonstrate that we get 100% white at the end!

Now insert a barrier in the s path:



What should we expect to emerge at *h and s*?

(1) 50% less electrons at *h and s*.

(2) What *Color*? ← *Experiments demonstrate that 50% are white and 50% are black!*

Recap:

Without barrier: 100% of electrons at *h and s* are *white*.

With barrier: 50% are white and 50% are *black*.

Big Question: *What path does an individual white electron take without the barrier present?*

(A) Path *h*? ***No!*** Electrons that take path *h* are hard, so their *Color* statistics are 50/50. But without the barrier, the *Color* statistics of electrons are 100% white.

(B) Path *s*? ***No!*** (Same reason.) Electrons that take path *s* are soft, so their color statistics are 50/50. But without the barrier, the *Color* statistics are 100% white.

(C) Both *h* and *s*? ***No!*** Experimentally, feed in an electron and check which path it takes. 50% will be on path *h*, and 50% will be on path *s*.

(D) Neither *h* nor *s*? ***No!*** Block both paths and nothing is registered at *h and s*.

SO: What can we say about a *white* electron in our device?

One option: We can say *Hardness* is an *indeterminate property* of *white* electrons. A *white* electron is in a *superposition* of being *hard* and *soft*.

or

A *white* electron just *is* an electron in a superposition of *hard* and *soft*.

> ***Moral:*** Quantum *properties* are very different from classical *properties*.
> So the *mathematical description* of quantum properties will have to be different from that of classical properties.

Let's get just a bit more general and then a bit more specific:

How to Describe Physical Phenomena: 5 Essential Notions

(a) ***Physical system***

Classical example: a baseball — *Quantum example:* an electron

(b) ***State** of a physical system*: *A description of the system at an instant in time in terms of its properties.*

Classical examples:
(i) A baseball moving at 95 *mph*, 5 feet from batter.
(ii) A baseball moving at 50 *mph*, dropping over left-field wall.

Quantum examples:
(i) An electron entering a Hardness box.
(ii) An electron exiting a Hardness box.

(c) ***Properties** of a physical system*

Classical examples:	Quantum examples:	
momentum	*hardness*	(spin along certain direction)
position	*color*	(spin along another direction)
energy	*momentum*	
	position	
	energy	

(d) ***State space** of a physical system*: *The collection of all possible states of a physical system.*

(e) ***Dynamics** of a physical system*: *Description of how the states of the system change with time.*

ex: Newtonian dynamics (Newton's equations of motion)
- describe how the states of a classical system change with time

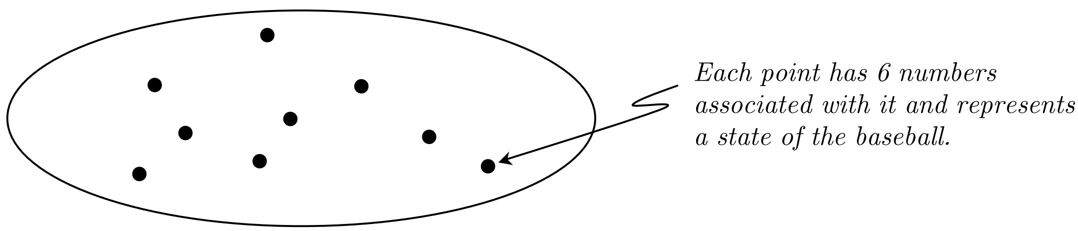
Given an initial state (baseball leaving the pitcher's hand with given momentum and position), what is the state of the baseball at a later time (as it crosses homeplate; as a given force is imparted on it; as it soars over the left-field wall)?

Mathematical Description of Classical Physical System

Baseball example:

(i) A ***state*** of the baseball is specified by giving the baseball's *momentum* (it's mass times its velocity) and its *position*. (Newton's equations of motion uniquely determine the way such states change in time.) To specify the baseball's momentum at any given time requires 3 numbers p_1, p_2, p_3 (its velocity has 3 components); to specify its position also requires 3 numbers q_1, q_2, q_3. (So the baseball has 6 "degrees of freedom".)

(ii) The ***state space*** of the baseball can thus be represented by a 6-dimensional set of points (called the "phase space" of the system):



(iii) All dynamical ***properties*** of the baseball can be represented by *functions* of its momentum p_i and position q_i ($i = 1, 2, 3$), and hence by functions on the phase space. These functions are in principle ***always well-defined**** for any point in phase space.

<u>*Ex*</u>: baseball's *energy* $= E(p_i, q_i) = (p_1^2 + p_2^2 + p_3^2)/2m$

baseball's mass (← m)

** Provided the phase space is well-behaved topologically, which is normally the case.*

(iv) Newton's equations of motion, which provide the ***dynamics*** for classical mechanics, can be reformulated as equations that act on states in phase space. The result is the *Hamiltonian formulation* of classical mechanics.

Will such a mathematical description work for quantum mechanics?

NO! We now know that quantum properties are *not* always well-defined. So we can't represent them mathematically as functions on a set of points.

What mathematical representation should we use?

In particular, we need to represent quantum properties, quantum states, and quantum state spaces.

It turns out that the following mathematical representations work:

	Classical mechanics	*Quantum mechanics*
state space	set of points (phase space)	vector space
states	points	vectors
properties	functions of points	operators on vectors

Now: Need to say what these mathematical objects are, how they represent quantum states and properties, and give some examples.