12a. Modal Interpretations

• Let's return to using Hilbert spaces to represent QM state spaces, and operators to represent properties.

• Recall: The *Kochen-Specker Theorem* says that the properties associated with a Hilbert space $\mathcal{H}$ can't *all* have values at the same time (if $\dim \mathcal{H} \geq 3$).

• One Way to Avoid KS: Claim that *some* (not all) properties defined on $\mathcal{H}$ always have determinate values (even in superpositions), others do not.

• Ex: Bohm's Theory
  ○ One property (position) is always determinate (always has a value).
  ○ All other properties are *contextual* -- their values depend on how they are measured.

---

**Modal Interpretations Claim:**

(A) For any Hilbert space $\mathcal{H}$, at any given time, there is a subset of properties that are always determinate (always possess values).

(B) The QM probabilities for these properties are *epistemic*: for these properties, probabilities represent our ignorance of their actual values.
Modal Interpretations reject the Eigenvector/Eigenvalue Rule.

\[
\begin{align*}
\text{The state } |\psi\rangle & \text{ possesses the value } \lambda \text{ of a property represented by an operator } O. \\
\text{if and only if} & \\
\text{The state } |\psi\rangle & \text{ is an eigenvector of } O \\
& \text{with eigenvalue } \lambda.
\end{align*}
\]

- Modal interpretations allow for a state to possess the value of a property, even when it is not an eigenvector of the associated operator.

- **They claim:** For any given state $|\psi\rangle$, in addition to those properties for which $|\psi\rangle$ is an eigenvector, there are other properties for which $|\psi\rangle$ also possesses values (the "modal", always-determinate properties).

- **So:** Modal interpretations agree with the "if" part ($\Leftarrow$) of EE.

- **But:** They disagree with the "only if" part ($\Rightarrow$) of EE.

- **Initial task for Modal Interpretations:** Identify the subset of determinate "modal" properties.

- There are different versions of modal interpretations, depending on how they identify this subset.

- At any given time, the subset of determinate properties is given by the basis states of the *biorthogonal expansion* of the system's state vector.

**Biorthogonal Decomposition Theorem:**

Let $|Q\rangle$ be a vector in the product Hilbert space $\mathcal{H}_1 \otimes \mathcal{H}_2$. Then there is a basis $|a_1\rangle$, ..., $|a_N\rangle$ of $\mathcal{H}_1$, and a basis $|b_1\rangle$, ..., $|b_N\rangle$ of $\mathcal{H}_2$ such that $|Q\rangle$ can be expanded as:

$$|Q\rangle = c_{11}|a_1\rangle|b_1\rangle + c_{22}|a_2\rangle|b_2\rangle + \ldots + c_{NN}|a_N\rangle|b_N\rangle$$

And, if $|c_{11}| \neq |c_{22}| \neq \ldots \neq |c_{NN}|$, then these bases are *unique*.

- In general, if $|g_1\rangle$, ..., $|g_N\rangle$ and $|h_1\rangle$, ..., $|h_N\rangle$ are bases of $\mathcal{H}_1$ and $\mathcal{H}_2$, then any vector $|Q\rangle$ can be expanded as

$$|Q\rangle = d_{11}|g_1\rangle|h_1\rangle + d_{12}|g_1\rangle|h_2\rangle + \ldots + d_{21}|g_2\rangle|h_1\rangle + d_{22}|g_2\rangle|h_2\rangle + \ldots$$

- The *Biorthog Decomp Theorem* says that there are some bases in which the "cross terms" with coefficients $d_{12}$, $d_{21}$, etc, all vanish. And these bases will be unique just when all the remaining coefficients are different from each other.
Why do we want bases in which the cross terms vanish?

- Because these are the bases associated with post-measurement systems.

- Consider a composite system composed of a Hardness measuring device $m$ and a black electron $e$.

- The basis for the composite $m$-$e$ system is given by

\[ \{|"hard"\rangle_m|hard\rangle_e, |"hard"\rangle_m|soft\rangle_e, |"soft"\rangle_m|hard\rangle_e, |"soft"\rangle_m|soft\rangle_e \} \]

- So the general expansion of a state $|Q\rangle$ is:

\[ |Q\rangle = a|"hard"\rangle_m|hard\rangle_e + c|"hard"\rangle_m|soft\rangle_e + d|"soft"\rangle_m|hard\rangle_e + b|"soft"\rangle_m|soft\rangle_e \]

- If this basis is biorthogonal, then $c = d = 0$, and we have:

\[ |Q\rangle = a|"hard"\rangle_m|hard\rangle_e + b|"soft"\rangle_m|soft\rangle_e \]

- This is just the post-measurement state of our composite system!

- We could avoid the Projection Postulate if we assume that the properties associated with these bases vectors (pointing to "hard" or "soft" for $m$, being hard or soft for $e$) are always determinate.

- So: KHD just stipulates that properties associated with biorthogonal expansion bases are always determinate.
**KHD Rules (replace Projection Postulate):**

**Rule 1:** For any physical system \( S \) that is composed of two subsystems \( S_1 \) and \( S_2 \), there are some properties for which \( S \) always possesses values. To identify them:

(i) Expand the state vector \( |Q\rangle \) for \( S \) in its biorthogonal decomposition:

\[
|Q\rangle = c_{11} |a_1\rangle |b_1\rangle + c_{22} |a_2\rangle |b_2\rangle + \ldots + c_{NN} |a_N\rangle |b_N\rangle
\]

(ii) The biorthogonal basis states \( |a_1\rangle, \ldots, |a_N\rangle \), and \( |b_1\rangle, \ldots, |b_N\rangle \) are the eigenvectors of the determinate properties, call them \( A \) and \( B \).

(iii) The subsystems \( S_1 \) and \( S_2 \) can be said to have determinate values for the properties \( A \) and \( B \), so identified.

**Rule 2:** (Born Rule) If \( S \) is in the state \( |Q\rangle \), then the probability that \( S_1 \) has the value \( a_i \) of the property \( A \) is \( c_{ii}^2 \), and the probability that \( S_2 \) has the value \( b_i \) of the property \( B \) is \( c_{ii}^2 \).
Why this is helpful:
• Suppose a system is in a state represented by

\[ |Q\rangle = a|\text{"hard"}\rangle_m|\text{hard}\rangle_e + b|\text{"soft"}\rangle_m|\text{soft}\rangle_e \quad (a \neq b). \]

• *A Literal Interpretation says*: This is a state in which \( e \) can't be said to have the Hardness property, and \( m \) can't be said to be indicating "hard" or "soft".

• *KHD says*: This is a state in which \( e \) does have a definite value of Hardness, and \( m \) is definitely pointing to either "hard" or "soft" (even though we don't know what Hardness value \( e \) has, and we don't know where \( m \) is pointing).

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**Essential Characteristics of Modal Interpretations**

(A) Rejection of Eigenvector/Eigenvalue Rule.

(B) Rejection of Projection Postulate.

(C) Probabilities are epistemic.
Problems with KHD

1. Non-uniqueness of biorthogonal expansions.
   - Consider the biorthogonal expansion $|Q⟩ = c_{11}|a_1⟩|b_1⟩ + \ldots + c_{NN}|a_N⟩|b_N⟩$.
   - The Biorthog Decomp Theorem says this expansion is unique, provided that $|c_{11}| \neq |c_{22}| \neq \ldots \neq |c_{NN}|$.
   - If this does not hold; i.e., if any of the expansion coefficients are equal, then there will be other biorthogonal expansions of $|Q⟩$, in fact infinitely many.

   Ex: $|Q⟩ = \sqrt{\frac{1}{2}}|\text{hard}\rangle_m|\text{hard}\rangle_e + \sqrt{\frac{1}{2}}|\text{soft}\rangle_m|\text{soft}\rangle_e$
   
   $= \sqrt{\frac{1}{2}}|\text{black}\rangle_m|\text{black}\rangle_e + \sqrt{\frac{1}{2}}|\text{white}\rangle_m|\text{white}\rangle_e$
   
   $= \text{etc.}$

   - In such cases, KHD Rule 1 will say that infinitely many properties will have definite values at any given time, and this violates the KS Theorem!
2. **Dynamics for determinate properties.**

- All modal interpretations (not just *KHD*) say that, at any given time, a physical system possesses the values of some subset of properties (in addition to, and including those given by the *EE Rule*).
- Let $Det_t$ be this set of determinate properties at time $t$. This set can change from moment to moment! In other words, $Det_t$ may be different from $Det_{t'}$ for $t \neq t'$.
- **Ex:** In the *KHD* version, $Det_t$ depends on the component states of the composite system, and these component states may change over time.
- **So:** All modal interpretations need to tell us how $Det_t$ changes over time. They need to give us a *dynamics* for the determinate properties. *KHD* does not specify this.

**Ex:** Bohmian Mechanics can be considered as a modal interpretation in which the property dynamics (for the position property) is given by Bohm's Equation.
3. **Imperfect Measurements.**

- **Claim**: The post-measurement states that KHD identifies represent *ideal* perfect measurements. For *actual* imperfect measurements, KHD does not pick out the right post-measurement properties.

- KHD seemed to work for the post-measurement state:

  $$|Q\rangle = a|h\rangle_m|h|_e + b|s\rangle_m|s|_e$$  
  (suppose $a \neq b$)

- This is in the form of a biorthogonal expansion, so KHD says: the electron has a definite value of Hardness.

- **But**: The Schrödinger-evolved post-measurement state will really be:

  $$|J\rangle = |hard\rangle_m|h|_e + |soft\rangle_m|s|_e + f|h\rangle_m|s|_e + g|s\rangle_m|h|_e$$

**Error terms!** Represent the fact that real measuring devices will never perfectly correlate pointers with Hardness property. For realistic measuring devices, $f$ and $g$ can be made very small, but they will never vanish.
3. **Imperfect Measurements.**

- **Claim:** The post-measurement states that *KHD* identifies represent *ideal* perfect measurements. For *actual* imperfect measurements, *KHD* does not pick out the right post-measurement properties.

- *KHD* seemed to work for the post-measurement state:

  $$|Q\rangle = a|"hard"\rangle_m|\text{hard}\rangle_e + b|"soft"\rangle_m|\text{soft}\rangle_e$$  
  (suppose $a \neq b$)

- This is in the form of a biorthogonal expansion, so *KHD* says: the electron has a definite value of *Hardness*.

- **But:** The Schrödinger-evolved post-measurement state will *really* be:

  $$|J\rangle = c|"hard"\rangle_m|\text{hard}\rangle_e + d|"soft"\rangle_m|\text{soft}\rangle_e + f|"hard"\rangle_m|\text{soft}\rangle_e + g|"soft"\rangle_m|\text{hard}\rangle_e$$

  - $|J\rangle$ has a biorthogonal expansion (guaranteed by the *Biorthog Decomp Theorem*), but it will *not* be the one that *KHD* cites:
    $$|J\rangle = k|w\rangle_m|\text{grump}\rangle_e + l|w'\rangle_m|\text{gromp}\rangle_e$$
  
  - *Grump* and *gromp* are values of *some* property (they are eigenvectors of *some* operator), but not *Hardness*.

- So the *KHD Rule 1* entails that, after a *Hardness* measurement, the electron is either *grump* or *gromp*, and not either *hard* or *soft*. 
12b. Quantum Logic

Motivation:

- When a physical system is in a state represented by a superposition, we can't use classical logic to describe the properties it possesses.
- Consider the state $|Q\rangle = a|\text{"hard"}_m\rangle|\text{hard}_e\rangle + b|\text{"soft"}_m\rangle|\text{soft}_e\rangle$.
- Recall: Under a literal interpretation, an electron in this state:
  (a) Can't be said to be hard.
  (b) Can't be said to be soft.
  (c) Can't be said to be both hard and soft.
  (d) Can't be said to be neither hard nor soft.
- Perhaps to make sense of such superposed states, we need to change our logic!

Goal:
To develop a quantum logic that will allow us to say meaningful things about the properties of states in superpositions.
1. Classical Properties and Classical Logic

- Classical mechanics represents properties in a certain way (as functions on a phase space), and it turns out that this way has a structure associated with it that can be identified with the structure of classical logic.

- We know that QM represents properties in a different way (as operators on a Hilbert space). So the structure of QM properties is different from that of CM properties, and hence, classical logic.

The Logic of Classical Mechanics

- **Recall:** CM state space = phase space (set of points)
  - CM states = points
  - CM properties = functions

- Consider the property, "The value of property $A$ is $a$".

- In QM, this property can be represented by a projection operator.

- In CM, this property is represented by a subset of phase space; namely, the collection of all phase space points that represent states in which the value of property $A$ is $a$. 
Phase space $\Omega$ and 3 subsets: $P$, $Q$, $R$.

Let $P$ represent the property "The value of property $A$ is $a$". This means that all points in $P$ represent states in which the value of property $A$ is $a$.

Let $Q$ represent the property "The value of property $B$ is $b$".

The intersection $P \cap Q$ represents the property "The value of property $A$ is $a$ and the value of property $B$ is $b$".

All properties that a physical system can possess can be represented by sentences of the sort "The value of property $X$ is $x$".

So subsets of points represent CM properties in general.
- Let the set $P$ represent the sentence $p = "The value of property A is a."$
- Let the set $Q$ represent the sentence $q = "The value of property B is b."$
- Let the set $R$ represent the sentence $r = "The value of property C is c."$

- $P \cap Q$ represents "The value of property A is $a$ and the value of property $B$ is $b$" (or "$p \land_c q$").
- $P \cup R$ represents "The value of property A is $a$ or the value of property $C$ is $c$" (or "$p \lor_c q$").
- $\neg P$ represents "The value of property A is not $a$." (or "$\neg_c p$").

<table>
<thead>
<tr>
<th>set operation</th>
<th>classical logic connective</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\cap$ (intersection)</td>
<td>$\land_c$ (and)</td>
</tr>
<tr>
<td>$\cup$ (union)</td>
<td>$\lor_c$ (or)</td>
</tr>
<tr>
<td>$\neg$ (complement)</td>
<td>$\neg_c$ (not)</td>
</tr>
</tbody>
</table>

- A collection of sets with $\cap$, $\cup$, $\neg$ defined on it and a collection of sentences with $\land_c$, $\lor_c$, $\neg_c$ defined on it are both representations of a Boolean algebra.
- CM properties, collections of sets, and classical logic all have the same Boolean algebraic structure.
• **So:** Classical logic = the logic of the structure of $CM$ properties.
• An *empirical* approach to logic!
• Why do we use classical logic to describe the world?
• Because of the way classical physics describes the world.
• This suggests that, when the physics changes, so should the logic!

*The Logic of Quantum Mechanics*

• **Recall:** $QM$ state space = Hilbert space $\mathcal{H}$
  
  $QM$ states = vectors
  
  $QM$ properties = operators

• Consider the property, "The value of property $A$ is $a$".
• Represented by a projection operator $P_{|a\rangle}$.
• $P_{|a\rangle}$ projects any vector onto the 1-dim *subspace* of $\mathcal{H}$ (i.e., ray) defined by the eigenvector $|a\rangle$ of $A$ with eigenvalue $a$.
• **So:** In $QM$, properties of the type "The value of property $X$ is $x$" are represented by *subspaces* (and not *subsets*).
2. The Structure of Quantum Properties

**Def:** A subspace of a Hilbert space $\mathcal{H}$ is a subset of $\mathcal{H}$ closed under vector addition and scalar multiplication.

- **This means:** A subspace is just a part of $\mathcal{H}$ that is itself a vector space.
- There is a 1-1 correspondence between projection operators and subspaces.

**Subspaces are related by 3 operations:**

- $\cap$ (intersection) \hspace{1cm} $V \cap W = \{\text{all vectors in both } V \text{ and } W\}$
- $\oplus$ (linear span) \hspace{1cm} $V \oplus W = \{\text{all linear combinations of vectors from } V \text{ and } W\}$
- $\perp$ (orthocomplement) \hspace{1cm} $V^\perp = \{\text{all vectors that are orthogonal to vectors in } V\}$

- If $V$ and $W$ are both 1-dim, then $V \oplus W$ is a 2-dim subspace -- a plane containing all vectors of the form $a|v\rangle + b|w\rangle$, where $|v\rangle \in V$ and $|w\rangle \in W$.
- $V \oplus W$ corresponds to the projection operator $P_{V\oplus W} = P_{|v\rangle} + P_{|w\rangle}$.
- If $V$ is 1-dim and $W$ is 2-dim, then $V \oplus W$ is a 3-dim subspace; etc.
2. The Structure of Quantum Properties

**Def:** A *subspace* of a Hilbert space $\mathcal{H}$ is a subset of $\mathcal{H}$ closed under vector addition and scalar multiplication.

- **This means:** A subspace is just a part of $\mathcal{H}$ that is itself a vector space.
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### Subspaces are related by 3 operations:

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- $\oplus$ (linear span) \quad $V \oplus W = \{\text{all linear combinations of vectors from } V \text{ and } W\}$
- $\perp$ (orthocomplement) \quad $V^\perp = \{\text{all vectors that are orthogonal to vectors in } V\}$

- **Why linear span replaces union:** The *union* of two subspaces is not in general a subspace.

  - Suppose $V$, $W$ are both 1-dim subspaces of $\mathcal{H}$.
  - Then $V \cup W$ is the set of all vectors in both $V$ and $W$.
  - This set is not a subspace: The sum of two vectors from $V$ and $W$ may not itself be in $V \cup W$ (it may point in a direction other than the directions defined by $V$ and $W$).
• The structure of QM properties is given by the *subspace structure* of a Hilbert space (as opposed to the *subset structure* of a phase space).

• Important property of the subspace structure: *It is not distributive:*

• **Claim:** For any subspaces V, W, X, of H, it is not in general the case that

\[ X \cap (V \oplus W) = (X \cap V) \oplus (X \cap W). \]

**Proof:**
• Suppose V, W and X are subspaces of H and suppose X is a subspace of V \( \oplus \) W such that X is neither a subset of V nor a subset of W.

• This means: Any vector |x⟩ in X can be written as |x⟩ = a|v⟩ + b|w⟩, with |v⟩ ∈ V and |w⟩ ∈ W and a, b nonzero.

• Then \( X \cap (V \oplus W) = X \).

• But \( (X \cap V) \oplus (X \cap W) = 0 \oplus 0 = 0. \)

• Since Boolean algebras are distributive, this means that the subspace structure of QM properties is not a Boolean algebra.

• So it really is different from the subset structure of CM properties and the structure of classical logic (which are Boolean)!
Now: Construct a non-Boolean quantum logic based on the following correspondences:

<table>
<thead>
<tr>
<th>subspace operation</th>
<th>quantum logic connective</th>
</tr>
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<tbody>
<tr>
<td>( \cap ) (intersection)</td>
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<td>( \lor_q ) (or)</td>
</tr>
<tr>
<td>( \perp ) (orthocomplement)</td>
<td>( \neg_q ) (not)</td>
</tr>
</tbody>
</table>

- Let the subspace \( V \) represent the sentence \( v = \"The value of property A is a.\" \)
- Let the subspace \( W \) represent the sentence \( w = \"The value of property B is b.\" \)
- Let the subspace \( X \) represent the sentence \( x = \"The value of property C is c.\" \)

- \( V \cap W \) represents "The value of property A is a and the value of property B is b" (or "\( v \land_q w \)").
- \( V \oplus W \) represents "The value of property A is a or the value of property C is c" (or "\( v \lor_q w \)").
- \( V^\perp \) represents "The value of property A is not a" (or "\( \neg_q v \)").
Why this is supposed to help: We can now claim that, as a matter of QL (Quantum Logic):

1. "A has a definite value" is a QL tautology (always a true statement), for all properties $A$.

Why?

"A has a definite value." means

\[
\begin{align*}
\text{"The value of property } A \text{ is } a_1, \text{ or } \\
\text{the value of property } A \text{ is } a_2, \text{ or } \ldots, \\
\text{or the value of property } A \text{ is } a_N."
\end{align*}
\]

means

\[
\begin{align*}
\text{"The state of the system lies in } V_1 \oplus \cdots \oplus V_N."
\end{align*}
\]

where $V_1, \ldots, V_N$ are the 1-dim subspaces spanned respectively by the eigenvectors $|a_1\rangle, \ldots, |a_N\rangle$ of $A$.

Note: $V_1 \oplus \cdots \oplus V_N = \mathcal{H}$, and it's always true that the state of a system lies in its state space $\mathcal{H}$.

So: As a matter of QL, all properties always have definite values at all times, even properties of measuring devices in superposed states!
2. Statements about incompatible properties possessing simultaneous values are contradictory (always false).

- **Why?** Suppose $A$ and $B$ are incompatible properties (i.e., *Hardness* and *Color*).

\[
\begin{align*}
\text{"Property } A \text{ has a value and property } B \text{ has a value."} \\
\text{means} \\
\text{"(The value of } A \text{ is } a_1 \text{ and the value of } B \text{ is } b_1) \text{ or} \\
\text{(the value of } A \text{ is } a_1 \text{ and the value of } B \text{ is } b_2) \text{ or ... or} \\
\text{(the value of } A \text{ is } a_2 \text{ and the value of } B \text{ is } b_1) \text{ or ... or} \\
\text{(the value of } A \text{ is } a_N \text{ and the value of } B \text{ is } b_N)."
\end{align*}
\]

\[
\text{means} \\
\text{"The state of the system lies in } (V_1 \cap W_1) \oplus (V_1 \cap W_2) \oplus \\
\cdots \oplus (V_2 \cap W_1) \oplus (V_2 \cap W_2) \oplus \cdots \oplus (V_N \cap W_N)."
\]

- **Note:** Since $V_i$ and $W_j$ are disjoint for any $i, j$, all the intersection terms vanish, and we're left with $0 \oplus 0 \oplus \cdots \oplus 0 = 0$, where 0 is the zero subspace (contains no vectors).

- **But:** The state of the system is *somewhere* in $\mathcal{H}$. So the statement that it is "nowhere" (i.e., in the zero subspace) is always false.
Essential Characteristics of QL Interpretation

(A) Rejects Eigenvector/Eigenvalue Rule
(B) Rejects of Projection Postulate
(C) Probabilities are epistemic

• All 3 characteristics are a result of the QL claim that all properties have determinate values at all times.

• Major Problem: If QL says all properties of a system have definite values at all times, this gets around the Measurement Problem, but it then runs up against the Kochen-Specker Theorem!
One Option: To say that every property always has a value is not to say that there is always a value that every property has.

Let \( V_1, V_2, ..., V_N \) and \( W_1, W_2, ..., W_N \) be the 1-dim subspaces spanned by the eigenvectors \( |a_1\rangle, |a_2\rangle, ..., |a_N\rangle \) and \( |b_1\rangle, |b_2\rangle, ..., |b_N\rangle \) of two operators \( A, B \).

Then \( W_i \cap ( V_1 \oplus V_2 \oplus ... \oplus V_N ) \) represents the sentence:

"The value of property \( B \) is \( b_i \) and property \( A \) has a definite value." \( (*) \)

And \( ( W_i \cap V_1 ) \oplus ( W_i \cap V_2 ) \oplus ... \oplus ( W_i \cap V_N ) \) represents the sentence:

"(The value of \( B \) is \( b_i \) and the value of \( A \) is \( a_1 \)) or
(the value of \( B \) is \( b_i \) and the value of \( A \) is \( a_2 \)) or ...
or (the value of \( B \) is \( b_i \) and the value of \( A \) is \( a_N \))." \( (**) \)

Which means: "The value of \( B \) is \( b_i \) and the value of \( A \) lies in \( \{ a_1, a_2, ..., a_N \} \)."

Which means: "The value of \( B \) is \( b_i \) and there is a value that \( A \) has."

Now: \( W_i \cap ( V_1 \oplus V_2 \oplus ... \oplus V_N ) \neq ( W_i \cap V_1 ) \oplus ( W_i \cap V_2 ) \oplus ... \oplus ( W_i \cap V_N ) \).

So: The sentences \( (*) \) and \( (**) \) do not mean the same thing!

Thus: To say that property \( A \) has a definite value is not to say that there is some definite value \( (a_1, a_2, ..., a_N) \) it has!
**Def.** A *disjunctive property* is a property that possesses a *disjunction* (\(a_1\) or \(a_2\) or \(a_3\) or ...) of individual values, any one of which the property cannot be said to possess.

- **Upshot:** To get around the *KS Theorem*, QL can claim that all quantum properties are disjunctive properties.

- **Lingering Problem:**
  - Under this view, QL is motivated by the desire to view properties realistically (*i.e.*, to be able to say that properties do have definite values at all times).
  - Does the notion of a disjunctive property really provide us with an adequate notion of property realism?