06. EPR & Bell Thought Experiments

*How Should Superpositions be Interpreted? Part 1.*

(A) *Literally*
- QM description is complete; probabilities are *ontic*.
- *Sample Claim:* The properties of a quantum system in a superposed state are *indeterminate* (do not possess values).

(B) *Non-literally*
- QM description is incomplete; probabilities are *epistemic*.
- *Sample Claim:* The properties of a quantum system are *determinate* (possess values) at all times, even when the system is in a superposed state.

- EPR pushes towards (B).
- Bell pushes back.
1. **EPR Thought Experiment.** *(Einstein, Podolsky, Rosen 1935)*

![Diagram of EPR experiment](image)

| Initial state | Final state: Call it |\(|\Psi\rangle\) |
|---------------|----------------------|-----------------|
| \(|\text{P1 & P2 in source}\rangle\) | \(\sqrt{\frac{1}{2}}|_{\text{spin-up}}\rangle_1|_{\text{spin-down}}\rangle_2 - \sqrt{\frac{1}{2}}|_{\text{spin-down}}\rangle_1|_{\text{spin-up}}\rangle_2\) |

- Final state \(|\Psi\rangle\) is an **entangled** 2-particle state.
- If measurement on P1 yields spin-up, then \(|\Psi\rangle\xrightarrow{\text{collapse}}|_{\text{spin-up}}\rangle_1|_{\text{spin-down}}\rangle_2\).

**Suppose we interpret superpositions literally.**

- **Then:** Before measurement, spin orientations of P1 and P2 in final state are both **indeterminate**.
- **But:** After a measurement of P1 that yields spin-up, P2 suddenly (instantaneously!) has a **determinate** value of spin-down!
- **And:** This is the case no matter how far apart P1 and P2 have traveled!
1. **EPR Thought Experiment.** (Einstein, Podolsky, Rosen 1935)

- Final state \(|\Psi\rangle\) is an entangled 2-particle state.
- If measurement on P1 yields spin-up, then \(|\Psi\rangle\overset{\text{collapse}}{\longrightarrow}|\text{spin-up}_1\rangle|\text{spin-down}_2\rangle\).

**Conclusion:**

- (A) *Either* adopt a literal interpretation of superpositions, and accept "non-locality"...
- (B) *Or* accept that QM is incomplete.

- Einstein, Podolsky & Rosen adopt (B): Superpositions should not be interpreted literally; in particular, properties always have determinate values.
2. Bell Thought Experiment.  (Bell 1964)

• If QM is incomplete, then perhaps a "Hidden Variables" description of quantum states and properties is possible in which properties are always determinate (possess values) at all times.

• Can we compare QM to such a Hidden Variables Theory?

• Yes!
**Set-Up:**
- D1 and D2 measure spin along 1 of 3 axes (V, R, L) oriented at 120° with respect to each other.
- D1 and D2 set so that they don't measure spin along the same axis.

**Question:** What is the probability that P1 and P2 have different spin orientations (one spin-up and the other spin-down)?

**Method 1 (Literal QM):**
Assume properties do not have definite values before measurement.

**Method 2 (Hidden Variables):**
- Assume properties always have determinate values.
- Assume measurement outcomes do not depend non-locally on each other.

- Bell's (1964) result: Methods 1 and 2 make different predictions!
- Clauser, et al. (198x) result: Experiments confirm Method 1's predictions!
Method 1 (Literal QM Prediction): 3 possible pre-measurement states.

\[ |\text{P1 & P2 in source}\rangle \xrightarrow{\text{Schrödinger evolution}} \sqrt{\frac{1}{2}} |\text{spin-up}_V\rangle_1 |\text{spin-down}_V\rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}_V\rangle_1 |\text{spin-up}_V\rangle_2 \]

\[ \text{OR} \quad \sqrt{\frac{1}{2}} |\text{spin-up}_R\rangle_1 |\text{spin-down}_R\rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}_R\rangle_1 |\text{spin-up}_R\rangle_2 \]

\[ \text{OR} \quad \sqrt{\frac{1}{2}} |\text{spin-up}_L\rangle_1 |\text{spin-down}_L\rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}_L\rangle_1 |\text{spin-up}_L\rangle_2 \]

**Technical Result:** How to relate states for spins along different axes \( z, z' \)

\[ |\text{spin-up on axis } z\rangle = \cos(\theta/2) |\text{spin-up on axis } z\rangle + \sin(\theta/2) |\text{spin-down on axis } z\rangle \]

where \( \theta = \text{angle between } z \text{ and } z' \)

- **Ex:** \[ |\text{spin-up}_R\rangle_2 = \cos(120^\circ/2) |\text{spin-up}_V\rangle_2 + \sin(120^\circ/2) |\text{spin-down}_V\rangle_2 \]

- **So:** \( \text{Pr}(P2 \text{ spin-up}_V, \text{ given } P2 \text{ spin-up}_R) = |\cos(120^\circ/2)|^2 = 1/4. \)

- **Thus:** \( \text{Pr}(P2 \text{ spin-up}_V, \text{ given } P1 \text{ spin-down}_R) = \text{Pr}(P2 \text{ spin-up}_V, \text{ given } P2 \text{ spin-up}_R) = 1/4. \)

- **So:** \( \text{Pr}(P1 \text{ and } P2 \text{ have different spin orientations}) = 1/4. \)
Method 2 (Hidden Variables Prediction): 8 possible pre-measurement states.

\[
\begin{align*}
(V+L+R+)_{1} & \\
(V-L-R-)_{2} & \\
(V-L+R+)_{1} & \\
(V+L-R-)_{2} & \\
(V-L+R+)_{2} & \\
(V-L-R+)_{1} & \\
(V+L+R+)_{2} & \\
(V-L-R+)_{2} &
\end{align*}
\]

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Prob different spin orientation: 1 1/3 1 1/3 1/3 1/3 1/3 1/3 1/3
Method 2 (Hidden Variables Prediction): 8 possible final states.

- **So:** \( \Pr(P_1 \text{ and } P_2 \text{ have different spin orientations}) \geq 1/3. \)

- **Note on Locality:** We assume that if, say, \( P_1 \) is \( V^+ \) and \( P_2 \) is \( L^- \), then a measurement of the system will produce these values. (Measuring \( P_1 \) as \( V^+ \) will not instantaneously affect \( P_2 \)'s value, and *vis-versa.*)

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<td>( V ) ( R )</td>
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<td>( L ) ( V )</td>
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<td>( L ) ( R )</td>
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<td>( R ) ( V )</td>
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<tr>
<td>( R ) ( L )</td>
<td>+ –</td>
</tr>
<tr>
<td>Prob different spin orientation</td>
<td>1</td>
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**Recap**

- **Literal QM Prediction:**
  \[
  \Pr(P_1 \text{ and } P_2 \text{ have different spin orientations}) = \frac{1}{4}.
  \]

- **Hidden Variables Prediction:**
  \[
  \Pr(P_1 \text{ and } P_2 \text{ have different spin orientations}) \geq \frac{1}{3}.
  \]

**Literal QM says:**  **Hidden Variables says:**

- In 1 out of 4 trials, on average, the spin orientations of P1 and P2 will differ.
- At the least, in 1 out of 3 trials, on average, the spin orientations of P1 and P2 will differ.

- Do many trials...
- ...result is always Literal QM prediction!
Current Options

Value Definiteness (VD)
The properties of a quantum system are determinate (possess values) at all times, even when the system is in a superposed state.

- **EPR say**: Either QM is incomplete, or QM is non-local.

- Options for advocates of incompleteness:
  1. Local Hidden Variables Theory based on VD.
  2. Non-local Hidden Variables Theory based on VD.

  *Bell says: NO! Conflicts with experiment!*

- But what about:
  1. Non-local Hidden Variables Theory based on VD.

- Is non-locality really so "spooky"?
**Why Non-Locality Isn’t All That Spooky:**

- **Recall:** EPR state is represented by

  \[ |A\rangle = \sqrt{\frac{1}{2}} |\text{spin-up}\rangle_1 |\text{spin-down}\rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}\rangle_1 |\text{spin-up}\rangle_2 \]

- If the *outcome* of a spin measurement on P1 is *spin-up*, then

  \[ |A\rangle \xrightarrow{\text{collapse}} |\text{spin-up}\rangle_1 |\text{spin-down}\rangle_2 \]

- **So:** The *outcome* of a spin measurement on P2 will be *spin-down*.

- **And:** If the *outcome* of a spin measurement on P1 is *spin-down*, then the *outcome* of a spin measurement on P2 will be *spin-up*.

- This means that the *outcome* of a measurement on P2 depends non-locally on the *outcome* of a measurement on P1 (and *vice-versa*).

- **But:** The *outcome* of a measurement on P2 does *not* depend on whether or not a measurement was performed on P1.
**Check:**

(1) Suppose a spin measurement is done on P2.
   ○ Then \( \Pr(P2 \text{ spin-up}) = 1/2 \) and \( \Pr(P2 \text{ spin-down}) = 1/2 \).

(2) Suppose a spin measurement is done on P1 and then another is done on P2.
   ○ Then \( \Pr(P1 \text{ spin-up}) = 1/2 \) and \( \Pr(P1 \text{ spin-down}) = 1/2 \).
   ○ If P1 does have spin-up, then P2 will have spin-down.
   ○ If P1 does have spin-down, then P2 will have spin-up.

• **Thus** The outcome of a measurement on P2 is *equally likely* to be spin-up or spin-down, *regardless* of whether or not a measurement was performed on P1!

• **Upshot:** Non-locality of outcome dependence can’t be used to send signals.

**Ex:** If we measure P2 *here* to have spin-down, then we know P1 *over there* has spin-up.

• But we *don't* know if P1 was already found to have spin-up: We don't know if P2's having spin-down *here* is a consequence of someone over there measuring P1 to have spin-up.

• **So:** Non-locality doesn't violate a prohibition on faster-than-light signalling that can be associated with Special Relativity.