06. EPR & Bell Thought Experiments


(A) Literally
- QM description is complete; probabilities are *ontic*.
- *Sample Claim*: The properties of a quantum system in a superposed state are *indeterminate* (do not possess values).

(B) Non-literally
- QM description is incomplete; probabilities are *epistemic*.
- *Sample Claim*: The properties of a quantum system are *determinate* (possess values) at all times, even when the system is in a superposed state.

- EPR pushes towards (B).
- Bell pushes back.
1. **EPR Thought Experiment.** (Einstein, Podolsky, Rosen 1935)

- **Final state** \( |\Psi\rangle \) is an **entangled** 2-particle state.
- If measurement on P1 yields spin-up, then \( |\Psi\rangle \xrightarrow{\text{collapse}} |\text{spin-up}\rangle_1 |\text{spin-down}\rangle_2 \).

**Suppose we interpret superpositions literally.**

- **Then:** Before measurement, spin orientations of P1 and P2 in final state are **both indeterminate.**
- **But:** After a measurement of P1 that yields spin-up, P2 **suddenly (instantaneously!)** has a **determinate** value of spin-down!
- **And:** This is the case no matter how far apart P1 and P2 have traveled!
1. **EPR Thought Experiment.**

(Einstein, Podolsky, Rosen 1935)

- Final state $|\Psi\rangle$ is an entangled 2-particle state.
- If measurement on P1 yields spin-up, then $|\Psi\rangle \xrightarrow{\text{collapse}} |\text{spin-up}\rangle_1 |\text{spin-down}\rangle_2$.

**Conclusion:**

(A) *Either* adopt a literal interpretation of superpositions, and accept "non-locality"...
(B) *Or* accept that QM is incomplete.

- Einstein, Podolsky & Rosen adopt (B): Superpositions should not be interpreted literally; in particular, properties always have determinate values.
2. Bell Thought Experiment. (Bell 1964)

- If $QM$ is incomplete, then perhaps a "Hidden Variables" description of quantum states and properties is possible in which properties are always determinate (possess values) at all times.
- Can we compare $QM$ to such a Hidden Variables Theory?
- **Yes!**
**Set-Up:**

- D1 and D2 measure spin along 1 of 3 axes (V, R, L) oriented at 120° with respect to each other.
- D1 and D2 set so that they don't measure spin along the same axis.

**Question:** What is the probability that P1 and P2 have different spin orientations (one spin-up and the other spin-down)?

**Method 1 (Literal QM):**
Assume properties do not have definite values before measurement.

**Method 2 (Hidden Variables):**
- **Determinateness:** Properties always have determinate values.
- **Locality:** Measurement outcomes do not depend non-locally on each other.

- Bell's (1964) result: Methods 1 and 2 make different predictions!
- Clauser, *et al.* (198x) result: Experiments confirm Method 1's predictions!
Method 1 (Literal QM Prediction): 3 possible pre-measurement states.

\[ |\text{P1 & P2 in source} \rangle \xrightarrow{\text{Schrödinger evolution}} \sqrt{\frac{1}{2}} |\text{spin-up}_V \rangle_1 |\text{spin-down}_V \rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}_V \rangle_1 |\text{spin-up}_V \rangle_2 \]

OR

\[ \sqrt{\frac{1}{2}} |\text{spin-up}_R \rangle_1 |\text{spin-down}_R \rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}_R \rangle_1 |\text{spin-up}_R \rangle_2 \]

OR

\[ \sqrt{\frac{1}{2}} |\text{spin-up}_L \rangle_1 |\text{spin-down}_L \rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}_L \rangle_1 |\text{spin-up}_L \rangle_2 \]

Technical Result: How to relate states for spins along different axes z, z'

\[ |\text{spin-up on axis } z \rangle = \cos(\theta/2) |\text{spin-up on axis } z' \rangle + \sin(\theta/2) |\text{spin-down on axis } z' \rangle \]

where \( \theta = \text{angle between } z \text{ and } z' \)

• Ex: \( |\text{spin-up}_R \rangle_2 = \cos(120^\circ/2) |\text{spin-up}_V \rangle_2 + \sin(120^\circ/2) |\text{spin-down}_V \rangle_2 \)

• So: \( \Pr(\text{P2 spin-up}_V, \text{ given } \text{P2 spin-up}_R) = |\cos(120^\circ/2)|^2 = 1/4. \)

• Thus: \( \Pr(\text{P2 spin-up}_V, \text{ given } \text{P1 spin-down}_R) = \Pr(\text{P2 spin-up}_V, \text{ given } \text{P2 spin-up}_R) = 1/4. \)

• So: \( \Pr(\text{P1 and P2 have different spin orientations}) = 1/4. \)
**Method 2 (Hidden Variables Prediction):** 8 possible pre-measurement states.

\[
\begin{align*}
\text{I} & : \begin{cases} (V+L+R+)_1 \\ (V-L-R-)_2 \end{cases} \\
\text{II} & : \begin{cases} (V+L+R-)_1 \\ (V-L-R+)_2 \end{cases} \\
\text{III} & : \begin{cases} (V-L-R-)_1 \\ (V+L+R+)_2 \end{cases} \\
\text{IV} & : \begin{cases} (V-L-R-)_1 \\ (V+L+R+)_2 \end{cases}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Device settings</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>(D1) (D2)</td>
<td>I</td>
</tr>
<tr>
<td>(V) (L)</td>
<td>++</td>
</tr>
<tr>
<td>(V) (R)</td>
<td>++</td>
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<tr>
<td>(L) (V)</td>
<td>++</td>
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<tr>
<td>(L) (R)</td>
<td>++</td>
</tr>
<tr>
<td>(R) (V)</td>
<td>++</td>
</tr>
<tr>
<td>(R) (L)</td>
<td>++</td>
</tr>
</tbody>
</table>

**Prob different spin orientation:**

|             | 1     | 1/3   | 1     | 1/3   | 1/3   | 1/3   | 1/3   | 1/3   |
Method 2 (Hidden Variables Prediction): 8 possible final states.

- So: \( \Pr(P1 \text{ and } P2 \text{ have different spin orientations}) \geq \frac{1}{3}. \)

- Note on Locality: We assume that if, say, P1 is \( V^+ \) and P2 is \( L^- \), then a measurement of the system will produce these values. (Measuring P1 as \( V^+ \) will not instantaneously affect P2's value, and \textit{vis-versa}.)
Recap

• Literal QM Prediction:

\[
\text{Pr}(P1 \text{ and } P2 \text{ have different spin orientations}) = \frac{1}{4}.
\]

• Hidden Variables Prediction:

\[
\text{Pr}(P1 \text{ and } P2 \text{ have different spin orientations}) \geq \frac{1}{3}.
\]

**Literal QM says:**
In 1 out of 4 trials, on average, the spin orientations of P1 and P2 will differ.

**Hidden Variables says:**
At the least, in 1 out of 3 trials, on average, the spin orientations of P1 and P2 will differ.

• Do many trials...
• ...result is always Literal QM prediction!
**Current Options**

**Value Definiteness (VD)**

The properties of a quantum system are determinate (possess values) at all times, even when the system is in a superposed state.

- **EPR say**: Either QM is incomplete, or QM is non-local.

- Options for advocates of incompleteness:
  1. Local Hidden Variables Theory based on VD.
  2. Non-local Hidden Variables Theory based on VD.

  *Bell says: NO! Conflicts with experiment!*

- But what about:
  1. Non-local Hidden Variables Theory based on VD.

- Is non-locality really so "spooky"?
**Why Non-Locality Isn't All That Spooky:**

- **Recall:** EPR state is represented by
  \[ |A\rangle = \sqrt{\frac{1}{2}} |\text{spin-up}\rangle_1 |\text{spin-down}\rangle_2 - \sqrt{\frac{1}{2}} |\text{spin-down}\rangle_1 |\text{spin-up}\rangle_2 \]

- If the *outcome* of a spin measurement on P1 is *spin-up*, then
  \[ |A\rangle \xrightarrow{\text{collapse}} |\text{spin-up}\rangle_1 |\text{spin-down}\rangle_2 \]

- **So:** The *outcome* of a spin measurement on P2 will be *spin-down*.
- **And:** If the *outcome* of a spin measurement on P1 is *spin-down*, then the *outcome* of a spin measurement on P2 will be *spin-up*.

- This means that the *outcome* of a measurement on P2 depends non-locally on the *outcome* of a measurement on P1 (and *vice-versa*).
- **But:** The *outcome* of a measurement on P2 does *not* depend on whether or not a measurement was performed on P1.
**Check:**

(1) Suppose a spin measurement is done on P2.
   - Then \( \Pr(P2 \text{ spin-up}) = 1/2 \) and \( \Pr(P2 \text{ spin-down}) = 1/2 \).

(2) Suppose a spin measurement is done on P1 and then another is done on P2.
   - Then \( \Pr(P1 \text{ spin-up}) = 1/2 \) and \( \Pr(P1 \text{ spin-down}) = 1/2 \).
   - If P1 does have *spin-up*, then P2 will have *spin-down*.
   - If P1 does have *spin-down*, then P2 will have *spin-up*.

- **Thus** The outcome of a measurement on P2 is *equally likely* to be spin-up or spin-down, *regardless* of whether or not a measurement was performed on P1!
- **Upshot:** Non-locality of outcome dependence can't be used to send signals.

- **Ex:** If we measure P2 *here* to have spin-down, then we know P1 *over there* has spin-up.
- But we *don't* know if P1 was already found to have spin-up: We don't know if P2's having spin-down *here* is a consequence of someone over there measuring P1 to have spin-up.

- **So:** Non-locality doesn't violate a prohibition on faster-than-light signalling that can be associated with Special Relativity.