A Concept of Emergence for EFTs

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1. How to Construct an EFT.
2. The EFT Intertheoretic Relation.
3. Emergence in EFTs.
4. Other Notions of Emergence.
5. Conclusion.
1. How to Construct an EFT

Given a "high-energy" Lagrangian $\mathcal{L}[\phi(x)]$:

(I) Identify and eliminate high-energy degrees of freedom.

- Choose a cutoff $\Lambda$ and decompose $\phi(x) = \phi_H(x) + \phi_L(x)$.
- Perform integration over $\phi_H(x)$:

$$Z = \int \mathcal{D}\phi_L \mathcal{D}\phi_H e^{i\int d^D x \mathcal{L}[\phi_L, \phi_H]} = \int \mathcal{D}\phi_L e^{i\int d^D x \mathcal{L}_{\text{eff}}[\phi_L]}$$

(II) Construct local operator expansion of $\mathcal{L}_{\text{eff}}[\phi_L(x)]$.

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0 + \sum_i g_i \mathcal{O}_i$$
1. How to Construct an EFT

**Characteristics**

1. $\mathcal{L}[\phi(x)]$ describes $\infty$ DOF, $\mathcal{L}_{\text{eff}}[\phi_L(x)]$ describes finite DOF.

2. $\mathcal{L}_{\text{eff}}[\phi_L]$ is formally distinct from $\mathcal{L}[\phi]$.

3. $\phi_L(x)$ is "dynamically" distinct from $\phi(x)$.

4. Relation between $\mathcal{L}_{\text{eff}}$ and $\mathcal{L}$ cannot be presented as a formal derivation.
   - *Step I:* Informal choice of cutoff and low-energy DOF.
   - *Step II:* Approximation procedure involving informal identification of symmetries.

• **But:** It *can* be informally characterized through concrete examples...
2. The EFT Intertheoretic Relation

**Example 1:** Superfluid Helium 3-A

- With respect to $T_c$, high-energy degrees of freedom are fermionic $^3He$ atoms arranged in Cooper pairs:

$$\mathcal{L} = \Psi^\dagger \{ i \partial_t - \left( \frac{\partial^2}{2m} + \mu \right) \} \tau_3 \Psi + \mathcal{L}_{int}[\Psi, \Delta]$$

- **Non-relativistic** Lagrangian density. (Schakel 1998)
- $\Psi$ encodes creation/annihilation operators for $^3He$ atoms.
- Order parameter $\Delta$ encodes $^3He-A$ Cooper pair interaction.
2. The EFT Intertheoretic Relation

**Example 1:** Superfluid Helium 3-A

- With respect to $T_c$, low-energy degrees of freedom are bosonic hydrodynamical sound waves $\phi(x)$:

\[
\mathcal{L}_{\text{eff}} = -n[\partial_t \phi + \frac{1}{2m} (\partial_i \phi)^2] + \rho[\partial_t \phi + \frac{1}{2m} (\partial_i \phi)^2]^2
\]

- *Non-relativistic* Lagrangian density. (Schakel 1998)
- $\phi$ encodes phase of order parameter.
- $n$ and $\rho$ are the fermion number density and density of states.
2. The EFT Intertheoretic Relation

**Example 1:** Superfluid Helium 3-A

- With respect to ground state, low-energy degrees of freedom are massless fermions coupled to a Maxwell field:

\[
\mathcal{L}_{\text{eff}} = \overline{\Psi} \gamma^\mu (\partial_\mu - qA_\mu) \Psi + \mathcal{L}_{\text{Max}}
\]  (3)


- \(\Psi\) encodes creation/annihilation operators for \(^3\)He atoms.

- \(\gamma\)-matrices are determined by a Lorentz-signature "metric" \(g^{\mu\nu}\) that encodes \(^3\)He-A Cooper pair degrees of freedom.

- \(qA_\mu\) encodes position of "Fermi points" in 4-momentum space.
2. The EFT Intertheoretic Relation

*Comparison*

\[
\mathcal{L} = \Psi^\dagger \left\{ i\partial_t - \left( \partial_i^2 / 2m + \mu \right) \right\} \tau_3 \Psi + \mathcal{L}_{\text{int}}[\Psi, \Delta] \quad (1)
\]

\[
\mathcal{L}_{\text{eff}} = -n[\partial_i \varphi + \frac{1}{2m} (\partial_i \varphi)^2] + \rho[\partial_i \varphi + \frac{1}{2m} (\partial_i \varphi)^2]^2 \quad (2)
\]

\[
\mathcal{L}_{\text{eff}} = \bar{\Psi} \gamma^\mu (\partial_\mu - qA_\mu) \Psi + \mathcal{L}_{\text{Max}} \quad (3)
\]

a. High-energy theory (1) is formally and dynamically distinct from low-energy EFTs (2) and (3).

- High-energy theory (1) is a non-relativistic QFT describing *fermionic* degrees of freedom.

- EFT of $T_c$ (2) is a non-relativistic QFT describing *bosonic* degrees of freedom.

- EFT of ground state (3) is a *relativistic* QFT.
2. The EFT Intertheoretic Relation

Comparison

\[ \mathcal{L} = \Psi^\dagger \{ i\partial_t - (\partial_i^2/2m + \mu) \} \tau_3 \Psi + \mathcal{L}_{\text{int}}[\Psi, \Delta] \] (1)

\[ \mathcal{L}_{\text{eff}} = -n[\partial_t \varphi + \frac{1}{2m} (\partial_i \varphi)^2] + \rho[\partial_t \varphi + \frac{1}{2m} (\partial_i \varphi)^2]^2 \] (2)

\[ \mathcal{L}_{\text{eff}} = \bar{\Psi} \gamma^\mu (\partial_\mu - qA_\mu) \Psi + \mathcal{L}_{\text{Max}} \] (3)

b. (1), (2) and (3) describe distinct physical systems:

- (1) describes non-relativistic fermionic \(^3\)He atoms.
- (2) describes non-relativistic bosonic sound waves.
- (3) describes relativistic fermions coupled to a Maxwell field.
2. The EFT Intertheoretic Relation

*Suggests:*

1. *Failure of law-like deducibility:* The laws of $\mathcal{L}_{\text{eff}}$ are not deducible consequences of the laws of $\mathcal{L}$.

2. *Ontological distinctness:* Degrees of freedom of $\mathcal{L}_{\text{eff}}$ are (typically) associated with physical systems that are distinct from the physical systems associated with the DOF of $\mathcal{L}$.

3. *Ontological dependence:* DOF of $\mathcal{L}_{\text{eff}}$ are exactly the low-energy DOF of $\mathcal{L}$. (Physical systems described by an EFT do not "float free" of those described by its high-energy theory.)
2. The EFT Intertheoretic Relation

Example 2: 2-dim Quantum Hall Liquid

- High-energy degrees of freedom are electrons coupled to an external magnetic field $A_i$ and a Chern-Simons field $a_\mu$:

\[
\mathcal{L} = -\psi^\dagger \{ \partial_i - ie(A_0 - a_0) \} \psi - \frac{1}{2m} \psi^\dagger \{ \partial_i - ie(A_i + a_i) \} \psi \\
+ \mu \psi^\dagger \psi + \vartheta \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda
\]  

- Non-relativistic Lagrangian density. (Schakel 1998)

- $\vartheta$ chosen so that electrons $\psi$ have an even number of magnetic fluxes ("composite" fermions).

- Quantum Hall Effect: $\sigma = v(e^2/h)$,

\[
v = \frac{\text{(# electrons)}}{\text{(# states per energy level)}} = \text{integer or fraction}
\]
2. The EFT Intertheoretic Relation

**Example 2:** 2-dim Quantum Hall Liquid

- "Low-energy" degrees of freedom of bulk liquid are two Chern-Simons fields:

\[
\mathcal{L}_{\text{eff}} = \vartheta \varepsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \vartheta' \varepsilon^{\mu\nu\lambda}(A_\mu + a_\mu) \partial_\nu(A_\lambda + a_\lambda) \quad (5)
\]

- *Topological quantum field theory.* (Schakel 1998)
- \(a_\mu, (A_\mu + a_\mu)\) are two Chern-Simons fields.
- \(\vartheta'\) chosen to produce integer QHE.
- An EFT of the Fractional QHE, but not a low-energy EFT.
2. The EFT Intertheoretic Relation

**Example 2:** 2-dim Quantum Hall Liquid

- Low-energy degrees of freedom of edge are bosonic hydrodynamical sound waves \( \phi(x) \):

\[
\mathcal{L}_{\text{eff-edge}} = \frac{1}{8\pi} \left\{ \left( \partial_t \phi \right)^2 - \left( \partial_x \phi \right)^2 \right\}
\]  

(6)

- *Relativistic* (1+1)-dim Lagrangian density.  (Wenn 1990)
2. The EFT Intertheoretic Relation

Comparison

\[ \mathcal{L} = -\psi^\dagger \{ \partial_t - ie(A_0 - a_0) \} \psi - \frac{1}{2m} \psi^\dagger \{ \partial_i - ie(A_i + a_i) \} \psi \]
\[ \quad + \mu \psi^\dagger \psi + \vartheta \epsilon^{\mu \nu \lambda} a_\mu \partial_\nu a_\lambda \]  \hspace{1cm} (4)

\[ \mathcal{L}_{\text{eff}} = \vartheta \epsilon^{\mu \nu \lambda} a_\mu \partial_\nu a_\lambda + \vartheta' \epsilon^{\mu \nu \lambda} (A_\mu + a_\mu) \partial_\nu (A_\lambda + a_\lambda) \]  \hspace{1cm} (5)

\[ \mathcal{L}_{\text{eff-edge}} = (1/8\pi) \{ (\partial_t \phi)^2 - (\partial_x \phi)^2 \} \]  \hspace{1cm} (6)

a. High-energy theory (4) is formally and dynamically distinct from EFTs (5) and (6):

- High-energy theory (4) is a non-relativistic QFT.
- EFT of bulk (5) is a topological QFT.
- Low-energy EFT of edge (6) is a relativistic QFT.
2. The EFT Intertheoretic Relation

Comparison

\[ \mathcal{L} = -\psi^\dagger \left\{ \partial_t - ie(A_0 - a_0) \right\} \psi - \frac{1}{2m} \psi^\dagger \left\{ \partial_i - ie(A_i + a_i) \right\} \psi \]
\[ + \mu \psi^\dagger \psi + \vartheta \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \]  

(4)

\[ \mathcal{L}_{\text{eff}} = \vartheta \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \vartheta' \varepsilon^{\mu\nu\lambda} (A_\mu + a_\mu) \partial_\nu (A_\lambda + a_\lambda) \]  

(5)

\[ \mathcal{L}_{\text{eff-edge}} = (1/8\pi) \{ (\partial_t \phi)^2 - (\partial_x \phi)^2 \} \]  

(6)

b. (4), (5) and (6) describe distinct physical systems:

- (4) describes non-relativistic composite electrons.
- (5) describes two topological Chern-Simons fields.
- (6) describes relativistic massless bosons.
3. Emergence in EFTs

Two General Notions of Emergence:

(a) Emergence as descriptive of the ontology (entities, properties) associated with a physical system with respect to another.

- To say phenomena associated with an EFT are emergent is to say the entities or properties described by the EFT emerge from those described by a high-energy theory.

(b) Emergence as a relation between theories.

- To say phenomena associated with an EFT are emergent is to say the EFT stands in a certain relation to a high-energy theory.
3. Emergence in EFTs

My Approach:

- Use the (informal) intertheoretic relation between an EFT and its high-energy theory to inform an ontological notion of emergence appropriate for EFTs.

- Thus: Emergence (under this view) is not a formal characteristic of theories; but rather an interpretation-dependent characteristic.
3. Emergence in EFTs

**Disiderata**

(i) Emergence should involve *microphysicalism*: The emergent system should ultimately be composed of microphysical systems that comprise the fundamental system and that obey the fundamental system's laws.

(ii) Emergence should involve *novelty*: The properties of the emergent system should not be deducible from the properties of the fundamental system.

• (i) and (ii) are underwritten in the EFT context by the elimination of degrees of freedom (DOF)...
3. Emergence in EFTs

How the properties of a system described by $\mathcal{L}_{\text{eff}}$ emerge from a fundamental system described by $\mathcal{L}$:

(i) *Microphysicalism:* High-energy DOF are integrated out of $\mathcal{L}$, which entails that the DOF of $\mathcal{L}_{\text{eff}}$ are exactly the low-energy DOF of $\mathcal{L}$.

(ii) *Novelty:* $\mathcal{L}_{\text{eff}}$ is expanded in a local operator expansion. The result is dynamically distinct from $\mathcal{L}$ in the sense of a failure of lawlike deducibility from $\mathcal{L}$ of the properties described by $\mathcal{L}_{\text{eff}}$. 
4. Other Notions of Emergence

(A) New Emergentism.

- **Claim (Mainwood 2006):** Microphysicalism and novelty characterize the "New Emergentism" of Anderson (1972) and Laughlin and Pines (2000).

- **But:** The mechanisms that underwrite New Emergentism are spontaneous symmetry breaking and universality.

- **And:** These mechanisms are typically *not* present in EFTs:
  - Present in EFTs for superfluid $^3$He-$A$.
  - Not present in EFTs for quantum Hall liquids.
4. Other Notions of Emergence


- **Claim:** Elimination of DOF plays two roles:

  (a) Secures the lawlike deducibility of an emergent entity's behavior from its composing parts (*physicalism*).

  (b) Entails that an emergent entity is characterized by different law-governed properties and behavior than those of its composing parts (*non-reductionism*).

- Applicable to EFTs?

- **No:** DOF elimination in an EFT is characterized by:

  (a) A *failure* of lawlike deducibility (*novelty*).

  (b) The retention, in the EFT, of the low-energy degrees of freedom of the high-energy theory (*microphysicalism*).
4. Other Notions of Emergence

(C) The Failure of a Limiting Relation.

• Necessary conditions for the existence of an emergent property described by a theory $T'$ with respect to a more fundamental theory $T$ (Batterman 2000):

(i) There must be a limiting relation between $T$ and $T'$.

(ii) The limiting relation must fail in the context in which the emergent property is identified; in particular, there must be a physical singularity associated with the emergent property.
4. Other Notions of Emergence

(C) *The Failure of a Limiting Relation.*

**Example (i):** Properties associated with phase transitions involving spontaneously broken symmetries. 

\[ T = \text{statistical mechanical description.} \]

\[ T' = \text{thermodynamical description.} \]

Limiting relation = \( N, V \rightarrow \infty, N/V = \text{const.} \)

- Limiting relation fails at a critical point/fixed point.
- Physical singularity = divergence in correlation length.
- Emergent properties = properties associated with the phase transition.
4. Other Notions of Emergence

(C) The Failure of a Limiting Relation.

**Example (ii):** Properties associated with a cutoff-regulated theory.

$T =$ renormalizable continuum theory.

$T' =$ cutoff-regulated theory.

Limiting relation $= \Lambda(s) \rightarrow \infty$, $[\text{bare parameters}] \rightarrow \infty$, $[\text{renormalized parameters}] = [\text{bare parameters}] / \Lambda(s) =$ const.

- Limiting relation fails at a fixed point (scale independence).
- Physical singularity $=$ divergence in Green's functions.
- Emergent properties $=$ properties associated with system at a fixed point.
4. Other Notions of Emergence

(C) The Failure of a Limiting Relation.

Example (ii): Properties associated with a cutoff-regulated theory.
$T = \text{renormalizable continuum theory.}$
$T' = \text{cutoff-regulated theory.}$
Limiting relation $= \Lambda(s) \to \infty$, $[\text{bare parameters}] \to \infty$,
$[\text{renormalized parameters}] = [\text{bare parameters}] / \Lambda(s) = \text{const.}$

- $T = \text{high-energy theory};$ $T' = \text{EFT}?$
- **No**: Not all EFTs are obtained from renormalizable high-energy theories.
- **Moveover**: $T$ and $T'$ are formally identical in Example (ii), whereas an EFT and its high-energy theory are not.
5. Conclusion

- Emergence in an EFT can be characterized by the elimination of DOF from a high-energy theory.

- This results in an EFT that can be interpreted as describing novel entities or properties in the sense of being dynamically independent of, and thus not deducible from, the entities or properties associated with a high-energy theory.

- These novel entities or properties can be said to ultimately be composed of the entities or properties that are constitutive of a high-energy theory (microphysicalism), insofar as the DOF exhibited by the former are exactly the low-energy DOF exhibited by the latter.